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TOOLS OF GAME THEORY IN ECONOMIC DECISION MAKING

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Abstrakt

Tato práce se zabývá současnými trendy v aplikaci teorie her k tvorbě ekonomických modelů, které se následně využívají při ekonomickém rozhodování s podporou prostředků informatiky. Práce se zejména opírá o poznatky teorie statických a dynamických her a her s dokonalými a nedokonalými informacemi. Zkoumány jsou modely týkající se sdílení zdrojů, aukcí a managementu. Pro každý z popsaných modelů je prezentována konkrétní aplikace.

Abstract

This thesis deals with the present trends in application of game theory to creation of economic models, which are subsequently used in economic decision making with the support of tools of information technology. It mainly focuses on the tools of static and dynamic games and games with perfect and imperfect information. Models of resource sharing, auctions and management are under investigation. For each of the described models a concrete application is presented.

Klíčová slova

Teorie her, Nashovo ekvilibrium, Bayesovské Nashovo ekvilibrium, statické hry, dynamické hry, modely oligopolů

Keywords

Game theory, Nash equilibrium, Bayesian Nash equilibrium, static games, dynamic games, Oligopoly models

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Tools of Game theory in economic decision making

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Poděkování

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Tato práce vznikla jako školní dílo na Vysokém učení technickém v Brně, Fakultě podnikatelské. Práce je chráněna autorským zákonem a její užití bez udělení oprávnění autorem je nezákonné, s výjimkou zákonem definovaných případů.

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Introduction

Imagine a classic childhood game of hide-n-seek. One child from the group is selected to be “it” and has to count to twenty with eyes closed at the base while the other kids hide. After counting, his task is to find the others and tag them. Hidden kids have to get to the base without getting tagged. Each one of them has one important decision to make – where to hide? Should I hide where I’ve been hidden before, or should I hide where the person that was “it” didn’t look before? Should I hide together with my friend? They have to consider other players’ decisions in order to make their own.

We have played games like these since we were children. What this game has in common with the games game theory analyzes is the process of making strategic decisions - decisions that are made in order to achieve a certain goal (win the game) while taking in consideration other players’ decisions. Game theory analyses these situations where multiple strategic decision makers interact together - they “play a game”.

In economic context we can consider bargaining about wage between a potential employee and employer as a game. Each player has a certain goal - the employee to get the job with the highest wage possible and employer to get the employee to work for him for the lowest wage possible. But both of them have to consider the other one when making an offer, the employer (if he wants the employee) cannot offer a wage that is too low, as the potential employer might get insulted and leave. Same goes for the potential employee - he cannot ask for a wage that is too high (if he wants the job). This is a game in the game theoretic sense.

Other example can be when a firm makes a decision whether to enter a new market. How will the firms that already are in this market react? Will they try to fight? Will the incumbent firm (firm that is already on the market) try to take preliminary measures to keep the potential competitors from entering the market? They are also playing a game. This work’s aim is to describe the models in game theory that are commonly applied to economic decision making. First, basic concepts of game theory and then basic games investigated by game theorists are reviewed. The following chapters describe more complex models and there is also an example of economic application for each one of them. A few applications using information technology are also presented.

1 Aim of the work

This work aims to describe and apply the current game theoretic models and methods on real world strategic economic decision problems. The first two chapters explain the most basic concepts and tools needed to solve more complicated situations. The following chapters describe in detail few of the most important theories together with their applications on more complex examples.

2 Theoretical basis of the work

This chapter contains basic concepts of game theory needed for further considerations and constructions.

2.1 Game

To analyze a game from a game theoretical point of view the game has to be clearly defined. We have to define **rules**, **players** and their available **strategies** and **payoffs** for each one of them. Based on the differences in the criteria above we can distinguish a number of different types of games.

2.1.1 Static games

Static games are the games which are the easiest to analyze. In static games each one of the players makes his decision simultaneously with the other players-they do not know the actions of other players at the point of making the decision. After making the decision the players are assigned payoffs based on the decisions chosen.

(GIBBONS, 1992, p.1)

A game of rock-paper-scissors can serve as an example for this type of games. Both players simultaneously choose between rock, paper and scissors and both of them instantly know who has won.

2.1.2 Dynamic games

In dynamic games, players move in a given order. When they are making their move, they are aware of actions made by players moving before them (but not necessarily all the moves). An example of a dynamic game can be the game of chess.

(BIERMAN and FERNANDEZ, 1998, p.121)

2.1.3 Games with incomplete information

Games in which one or both of the players has incomplete information about the payoffs are called games with incomplete information. An economic example can be a sale of drilling rights for a parcel. The buyers don't know if there really is a source of oil in this place or more precisely they don't know exactly how rich this source is.

Another example for this kind of games can be buying a used car. The car salesman knows the true value of the car he is selling, but the buyer does not. This is also an example of a game with **asymmetric information**.

2.1.4 Cooperative games

In cooperative games the players cooperate together to achieve their individual goals. The key in analyzing these games is to find the players' motivation to cooperate. Usually it is not simple to model cooperation - see Prisoner's dilemma in the next chapter.

2.2 Players

Players are the strategic decision makers in our games. In the wage-bargaining example in the introduction the players were the employee and the employer. Important notion is that the players are always considered **rational** – they always try to take actions which lead to the best results for them (they are selfish).

2.3 Strategies

A strategy in game theory represents a plan of response to each one of the courses the game can take. Strategy is a complete plan of actions for the game for the player. A list of strategies, one for each player, is called a **strategy profile**.

(BIERMAN and FERNANDEZ, 1998, p.7)

We can distinguish two kinds of strategies based on the presence of random factor:

- Pure strategies - they are not random
- Mixed strategies - they involve a probability for each move

Strategies will be capitalized - for example Buy, Don't buy. Strategy profiles will be written in curly braces – {Buy, Don't buy}, where Buy is the strategy for the first player and Don't buy is the strategy for the second player.

2.4 Payoffs

Payoff is the outcome that the player gets for each of the strategy profiles. They represent what the player gets at the end of the game. In this work the payoffs will be usually given in terms of monetary gains/losses. In real world people usually consider

more than just monetary gains. List of payoffs for each strategy profile is called the **payoff vector**. Payoffs for simple non-cooperative games can be represented using a matrix, for example the payoff matrix for the rock-paper-scissors game is depicted in table 2.1.

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	(0, 0)	(-1, 1)	(1, -1)
	Paper	(1, -1)	(0, 0)	(-1, 1)
	Scissors	(-1, 1)	(1, -1)	(0, 0)
Payoffs: (Player 1, Player 2)				

Table 2.1 Payoffs for the rock-paper-scissors game

Where payoff (0, 0) means that both players don't get anything – it's a draw, (1, -1) means that the first player has won and gets a payoff of 1 (1€ for example), (-1, 1) means that the second player has won and gets a payoff of 1.

2.4.1 Zero-sum games and non-zero sum games

A zero-sum game is the game where what one player loses, the other one wins. In other words the sum of the payoffs of all players is zero. On the other hand, games where the sum of payoffs is a non-zero value are called non-zero sum games.

2.5 Normal form

The normal-form representation of a n-player game specifies the players' strategy profiles S_1, \dots, S_n and their payoff vectors u_1, \dots, u_n . We can denote this game by $G = \{S_1, \dots, S_n; u_1, \dots, u_n\}$.

(GIBBONS, 1992, p.4)

Normal form representation can be used to describe a static game. It is also called the **strategic form** of the game.

3 Problem analysis and current situation

This chapter describes some of the classic games and explains how to solve them. These games and tools used to solve them will serve as a basis to understand and solve more complex games in the following chapters. Most important concepts introduced in this chapter are – game matrix, Nash equilibrium and Focal point equilibrium (Schelling point).

3.1 Solving a simple game with one player

In this subchapter, a simplified example game will be used to help explain the necessary tools needed to solve the following games. The game goes as follows:

We have a mid-sized lake in the mountains. In this lake a special kind of pearls of superb quality was found. These pearls do not regenerate, and it is assumed that there are 4000 of them in the lake. Each one of the pearls is worth 500€

The Pearl Hunters company has an exclusive pearl hunting 4-year license for this lake. The price of pearls does not change over the course of these 4 years. The company has to make a decision whether to buy a special robot which can search for the pearls. Without it, using only trained divers, they can expect to find 1000 pearls per year, so they will find all of the pearls in 4 years. If they buy the robot, they can expect that they will find 4000 pearls per year, which means they will find all of them in one year. This special robot costs 750 000€, while the training costs for divers are only 400 000€ The cost to extract the pearls is 100€per pearl. The following table sums the expected costs and revenues.

	Divers	Robot
Searching cost	400 000€	750 000€
Extracting cost	400 000€	400 000€
Total cost	800 000€	1 150 000€
Revenue	2 000 000€	2 000 000€
Profit	1 200 000€	850 000€

Table 3.1 Sum of costs, revenues and profits for the Pearl Hunters company

The player in this game is the Pearl Hunters company, its strategy is either to use the Divers or to use the Robot and the payoffs are respectively 1 200 000€ and 850 000€
In this example we can say without the use of any additional tools that the most profitable strategy is to not buy the Robot and use the Divers, getting the payoff of 1 200 000€

3.2 Game with two players

Now, we will consider one important modification to the previous game. The Pearl Hunters company does not have an exclusive license for the pearl hunting for the lake. There is another company that wants to invest in pearl hunting in this lake – Pearl Seekers. This company is in every aspect the same as the Pearl Hunters. But now there is one important difference in profits, which are split between the two companies. Both of the companies share the resources - pearls in the lake. In this game we assume that money in the future has the same value for the company as money today.

Now we have two players – Pearl Hunters and Pearl Seekers, both of them have two available strategies – Divers and Robot. So now we have four available strategy profiles:

- {Divers, Divers} – both companies use the Divers strategy
- {Divers, Robot} – Pearl Hunters company uses the Divers strategy, and the Pearl Seekers company uses the Robot strategy
- {Robot, Divers} – Pearl Hunters company uses the Robot strategy, and the Pearl Seekers company uses the Divers strategy
- {Robot, Robot} – both companies use the Robot strategy

Each one of the strategy profiles leads to a unique payoff. The payoffs for each strategy profile can be found in the following table.

		Pearl Seekers	
		Divers	Robot
Pearl Hunters	Divers	(400 000€, 400 000€)	(-30 000€, 480 000€)
	Robot	(480 000€, -30 000€)	(50 000€, 50 000€)
Payoffs: (Pearl Hunters, Pearl Seekers)			

Table 3.2 Payoff matrix for the Pearl Hunters and Pearl Seekers companies

The players in this game make their decisions simultaneously without knowing the decision of the other player - it is a **static game**. Now we will try to find the best decision that each player can make.

First we will analyze the situation from the Pearl Hunters' point of view. They do not know the decision the Pearl Seekers make, but they have to take it into account. If Pearl Hunters believe that Pearl Seekers will play the Robot strategy than it is reasonable to also play the Robot strategy, because it yields the payoff of 50 000€ compared to a payoff of -30 000€ if they would play the Divers strategy. On the other hand if Pearl Hunters believe that Pearl Seekers will play the Divers strategy, than the best they can do is to play the Robot strategy again and get a payoff of 480 000€ compared to payoff of 400 000€. So we can say that playing the Robot strategy **strictly dominates** playing the Divers strategy, or that the Divers strategy is **strictly dominated** by the Robot strategy. It means that it is never **rational** to play the Divers strategy in this game. Formally:

“The strategy S_1 **strictly dominates** the strategy S_2 for a player if, given any collection of strategies, that could be played by the other players, playing S_1 results in a strictly higher payoff for that player than does playing S_2 . The strategy S_2 is also said to be **strictly dominated** by S_1 . A rational player will never adopt a strictly dominated strategy nor expect a rational opponent to adopt one.”

(BIERMAN and FERNANDEZ, 1998, p.9)

At this point we can start looking for strictly dominated strategies in our game and eliminate them. We can eliminate the Divers strategy, as this one will never be played. This leaves us with only one strategy left - Robot. This strategy is **strictly dominant**.

“A **strictly dominant strategy** for a player is one that strictly dominates every other strategy of that player. A rational player will adopt a strictly dominant strategy whenever it exists”.

(BIERMAN and FERNANDEZ, 1998, p.10)

Because our players' payoffs are symmetrical, we now know that Pearl Seekers will also play the Robot strategy. We have only one strategy left for each player. The game is solved. The solution is a **strictly dominant strategy equilibrium**.

“The strategy profile (S_1, \dots, S_n) is a **strictly dominant strategy equilibrium** if for every player i , S_i is a strictly dominant strategy”.

(BIERMAN and FERNANDEZ, 1998, p.10)

But this equilibrium solution has one problem. The players are much worse off playing these equilibrium strategies than if they both play the Divers strategy. If both players can cooperate and play the Divers strategy they can both have payoffs of 400 000€ - 350 000€ more than when both playing the Robot strategy. The equilibrium outcome is **Pareto dominated**.

“The outcome O of a game is Pareto dominated if there is some other outcome O' such that:

- (1) every player either strictly prefers O' to O or is indifferent between O' and O ,
and
- (2) some player strictly prefers O' to O .

An outcome is Pareto optimal if it is not Pareto dominated by any other outcome of the game”.

(BIERMAN and FERNANDEZ, 1998, p.12)

To give a more complete list of basic definitions we will also define **weak dominance**:

“The strategy S_1 **weakly dominates** the strategy S_2 for a player if, given any collection of strategies that could be played by the other players, playing S_1 never results in a lower payoff to that player than does playing S_2 and, in at least one instance, S_1 gives the player a strictly higher payoff than does S_2 . The strategy S_2 is said to be **weakly dominated** by S_1 . A rational player will seldom play a weakly dominated strategy.

A **weakly dominant strategy** for a player is one that weakly dominates every other strategy of that player. A rational player will usually play a weakly dominant strategy.

The strategy profile (S_1, S_2, \dots, S_n) is a **weakly dominant strategy equilibrium** if for every player i , S_i is a weakly dominant strategy”.

(BIERMAN and FERNANDEZ, 1998, p.12)

If we slightly modify the previous game and add the option to use neither Divers nor the Robot we will get a different payoff scheme:

		Pearl Seekers		
		Divers	Robot	Nothing
Pearl Hunters	Divers	(400 000€ 400 000€)	(-30 000€ 480 000€)	(1 200 000€ 0€)
	Robot	(480 000€ -30 000€)	(50 000€ 50 000€)	(850 000€ 0€)
	Nothing	(0€ 1 200 000€)	(0€ 850 000€)	(0€ 0€)
		Payoffs: (Pearl Hunters, Pearl Seekers)		

Table 3.3 Payoffs for the pearl-hunting game with 3 strategies

In this modified game dominant strategy (weak or strong) no longer exists. Robot dominates Nothing, but it no longer dominates Divers. Divers and Nothing do not dominate any other strategy. But from the payoffs it's easy to see that the Nothing strategy will never be used, as the players can guarantee themselves at least the outcome of using Robot, which is at least 50 000€, which is better than 0€. The Nothing strategy is strictly dominated so we can eliminate it. When we eliminate the Nothing strategy we are left with the same result as before – to play the Robot strategy. This strategy is called **iterated strictly dominant**.

“A strategy is iterated strictly dominant for player i if and only if it is the only strategy in $-S_i$, where $-S_i$ is the intersection of the following sequence of nested sets of strategies:

- (1) S_i^1 consists of all of player i 's strategies that are not strictly dominated
- (2) For $n > 1$, S_i^n consists of the strategies in S_i^{n-1} that are not strictly dominated when we restrict the other players $j \neq i$ to the strategies in S_j^{n-1} .

The strategy profile (S_1, S_2, \dots, S_n) is an **iterated strictly dominant strategy equilibrium** if for every player i , S_i is an iterated strictly dominant strategy”. (BIERMAN and FERNANDEZ, 1998, p.14)

Iterated weakly dominance is a bit more complicated and will not be used in this work.

In our example we needed just one iteration to remove the Nothing strategy. In more complex games, there can be much more iterations.

In most of the games there will be more than one iterated strategy equilibrium. Example for such a game can be a modified pearl hunting game. The payoff scheme of this modified game is shown in table 3.4.

		Pearl Seekers		
		Divers	Robot	Nothing
Pearl Hunters	Divers	(400 000€, 400 000€)	(60 000€, 480 000€)	(1 200 000€, 0€)
	Robot	(480 000€, 60 000€)	(50 000€, 50 000€)	(850 000€, 0€)
	Nothing	(0€, 1 200 000€)	(0€, 850 000€)	(0€, 0€)
		Payoffs: (Pearl Hunters, Pearl Seekers)		

Table 3.4 Payoffs for the modified pearl-hunting game with 3 strategies

The difference is that when one of the companies uses the Divers strategy and the other one uses the Robot, they now don't get a loss of 30 000€, but instead they make a profit of 60 000€

Again, the Nothing strategy is strictly dominated for both Pearl Hunters and Pearl Seekers, as both firms can make more than 0€ in all cases using either the Divers or Robot strategy. So the Nothing strategy can be eliminated. The following table shows the payoff scheme after the elimination of dominated strategy.

		Pearl Seekers	
		Divers	Robot
Pearl Hunters	Divers	(400 000€, 400 000€)	(60 000€, 480 000€)
	Robot	(480 000€, 60 000€)	(50 000€, 50 000€)
		Payoffs: (Pearl Hunters, Pearl Seekers)	

Table 3.5 Payoffs for the modified pearl-hunting game with 3 strategies without the dominated strategies

Now there are no dominated strategies to eliminate. Both of the players are assumed to be rational, so they will always play the strategy which has the best outcome for them. If Pearl Hunters believe that Pearl Seekers will use Robot, then they have two options:

- Use Robot too and earn 50 000€
- Use Divers and earn 60 000€

The rational choice for Pearl Hunters is to use Divers (play the Divers strategy) and earn 60 000€. The Divers strategy is the **best response** to the Robot strategy being played by

Pearl Seekers. Now if Pearl Seekers believe that Pearl Hunters will use Divers they have two options too:

- Use Divers too and earn 400 000€
- Use Robot and earn 480 000€

The rational choice for Pearl Seekers now is to use Robot and earn 480 000€ which is a best response to the Divers strategy being played by the Pearl Hunters. The result is that players will adopt the strategy profile {Robot, Divers}. This belief is **self-confirming**, meaning that it is the rational choice for both players and neither one of them can do any better by unilaterally changing their choice. Because the payoff scheme is symmetrical, the same is true for the strategy profile {Divers, Robot}. This is the concept of the **Nash equilibrium** named after Professor John C. Nash. The formal definition is:

“Suppose there are N players in a game, X_i is the set of possible strategies for player i , and $v_i(s_1, \dots, s_N)$ is player i 's payoff when the players choose the strategy profile $\{s_i, \dots, s_N\}$. A **Nash equilibrium** is a strategy profile $\{s_i^*, s_N^*\}$ such that each strategy s_i^* is an element of X_i , and maximizes the function $f_i(x) = v_i(s_i^*, \dots, s_{i-1}^*, x, s_{i+1}^*, \dots, s_N^*)$ among the elements of X_i . That is, at a Nash equilibrium, each player's equilibrium strategy is a **best response** to the belief that the other players will adopt their Nash equilibrium strategies”.

(BIERMAN and FERNANDEZ, 1998, p.16)

3.3 Important simple game examples

3.3.1 Prisoner's dilemma

Prisoner's dilemma is a classic game whose result is not Pareto optimal.

The story goes:

“Two suspects are arrested and charged with a crime. The police lack sufficient evidence to convict the suspects, unless at least one confesses. The police hold the suspects in separate cells and explain the consequences that will follow from the actions they could take. If neither confesses then both will be convicted of a minor offense and sentenced to a one month in jail. If both confess then both will be sentenced to jail for six months. Finally, if one confesses but the other does not, then the confessor will be

released immediately but the other will be sentenced to nine months in jail – six for the crime and further three for obstructing justice”.

(GIBBONS, 1992, p.2-3)

		Prisoner 2	
		Cooperates	Defects
Prisoner 1	Cooperates	$(-1, -1)$	$(-9, 0)$
	Defects	$(-9, 0)$	$(-6, -6)$
Payoffs: (Prisoner 1, Prisoner 2)			

Table 3.6 Payoffs for the prisoner’s dilemma

Another example for Prisoner’s dilemma can be a Marketing war:

We have two companies and both of them have two strategies: either invest in an expensive advertising campaign or not. If both of them invest in this campaign, they both don’t get any competitive advantage over the other and end up only losing the money for the campaign. If only one of the companies pays for the expensive campaign, they get a larger market share and are more profitable and the second one gets a smaller share. If both of them don’t invest in the campaign, they don’t gain any additional market share, but they also don’t lose money for the campaign.

Payoff matrix for the advertising game is in table 3.7.

		Company 2	
		Invest	Don't invest
Company 1	Invest	$(0, 0)$	$(10, 0)$
	Don't invest	$(0, 10)$	$(3, 3)$
Payoffs: (Company 1, Company 2)			

Table 3.7 Payoffs for the advertising war game

The pearl hunting example game was also of this type.

To solve this game, we can search for a **Nash equilibrium**:

If Prisoner 1 believes that Prisoner 2 will cooperate he has two options:

- Defect and don’t spend any time in prison

- Cooperate and spend a month in prison

The rational choice is to defect and not go to jail. Now if Prisoner 2 believes that Prisoner 1 will defect, he has two options:

- Defect and get 6 months in prison
- Cooperate and get 9 months in prison

His best response is to defect, so strategy profile {Defect, Cooperate} is not a Nash equilibrium and neither is {Cooperate, Cooperate}.

Now suppose that Prisoner 1 believes that Prisoner 2 will defect. He has two options:

- Defect and get 6 months in prison
- Cooperate and get 9 months in prison

Rationally he will choose to defect and because the payoffs in the game are symmetrical, Player 2 will defect too. In conclusion, there is only one Nash equilibrium in this game, the strategy profile {Defect, Defect} – both players will defect and spend 6 months in jail (both will invest in the ad campaign and make zero profit). This outcome is Pareto dominated by the {Cooperate, Cooperate} strategy profile's outcome, which is one of the reasons why is Prisoner's dilemma one of the most known and studied game of game theory.

3.3.2 Coordination game/stag hunt

The coordination game goes like this:

You want to go on a date with your girlfriend to a classy restaurant. Your girlfriend prefers going to this restaurant dressed in formal clothing. You feel the same way. Most important is that both of you go to the restaurant dressed the same way.

The payoffs are represented in the following table.

		Your girlfriend	
		Formal	Casual
You	Formal	(2, 2)	(0, 0)
	Casual	(0, 0)	(1, 1)
Payoffs: (You, Your girlfriend)			

Table 3.8 Payoffs for the coordination game

This game is usually called the **coordination game**, as both players have to coordinate their moves to get the best result. In this instance it is going to the restaurant formally dressed.

Another variant of this game is called the **stag hunt**:

Two hunters go on a hunt. They both have two options:

- Hunt a stag
- Hunt a hare

They both make their decisions simultaneously and independently of each other. If they want to hunt the stag, they need to cooperate. They can hunt a hare alone, but the hare is worth less than a stag. The payoff scheme is the same as in the previous example.

There are two Nash equilibria:

- Both dress formally/Both go stag hunting
- Both dress casually/Both go hare hunting

But for the players to choose the Pareto optimal one – Formal/Stag we have to make a few assumptions about the game. We have to assume that:

- Both players know the payoff scheme – it is **common knowledge**
- Both players are **rational** – they try to maximize their payoffs given their knowledge about the game
- Both players prefer formal to casual/stag to hare

3.3.3 Battle of the sexes

This game is similar to the coordination game, but there is a slight modification. This time you want to go on a date with your girlfriend to a medium-class restaurant. Your girlfriend prefers going to this restaurant dressed in formal clothing. But you prefer going there dressed casually. But again, both of you prefer going to the restaurant dressed the same way to dressing differently.

The payoffs are represented in the following table.

		Your girlfriend	
		Formal	Casual
You	Formal	(1, 2)	(0, 0)
	Casual	(0, 0)	(2, 1)
Payoffs: (You, Your girlfriend)			

Table 3.9 Payoffs for the Battle of the sexes game

This game is usually called the Battle of the sexes. There are two Nash equilibria in this game: {Formal, Casual} and {Casual, Formal}. Without using threats/promises or changing the game there is no single solution to this game (no way to select one of the Nash equilibria), because any kind of reasoning that leads you to choose Casual clothing will lead your girlfriend to choose Formal, leading to the payoff scheme (0, 0).

Another way of solving the problem is to use the concept of **focal point** (or Schelling point), introduced by Thomas Schelling. A focal point is a Nash equilibrium (in games that have them) that somehow stands out as the best solution. As Schelling states it:

“Most situations -- perhaps every situation for people who are practiced at this kind of game -- provide some clue for coordinating behavior, some **focal point** for each person’s expectation of what the other expects him to expect to be expected to do. Finding *the* key, or rather finding *a* key -- any key that is mutually recognized as the key becomes *the* key -- may depend on imagination more than on logic; it may depend on analogy, precedent, accidental arrangement, symmetry, aesthetic or geometric configuration, casuistic reasoning, and who the parties are and what they know about each other”.

(SCHELLING, 1970, p.57)

An example of a focal point can be:

You have been to a few dates with your girlfriend and she always dresses formally. So it is safe to assume that she will do it this time again and you should do the same. So the strategy profile {Formal, Formal} is a **Schelling point** in this example.

3.3.4 Chicken

The game goes like this:

There are two drivers going against each other on a narrow road – you and your opponent. The one that swerves is called the **chicken**. There are four possible outcomes for this game:

- The opponent swerves, he is the chicken, you are the winner
- You swerve, you are the chicken, he is the winner
- Both of you swerve, both of you are chickens
- None of you swerve, cars crash, both of you are dead

The payoffs can be represented with the following table:

		The opponent	
		Don't swerve	Swerve
You	Don't swerve	$(-5, -5)$	$(2, -1)$
	Swerve	$(-1, 2)$	$(0, 0)$
Payoffs: (You, Your opponent)			

Table 3.10 Payoffs for the game of Chicken

There can be many variants for this game, for example:

A company has got into problems. The unionized workers are demanding salary raises, threatening they will go on a strike. If the management gives in to their demands, it loses its power (and money). If the workers give up, the union loses its power (and also money). If they both give in, nobody gains anything. But if both sides refuse to give in, there will be a strike, which is the worst outcome for both sides (same as crash in the previous example).

The Nash equilibria in this example are the ones where one of the players swerves/gives in and the other one does not. The profile {Swerve, Swerve} is not a Nash equilibrium, because if any of the players knew that the other one is going to swerve, he will choose not to swerve and “win” the game. But there is no simple way to decide which Nash equilibrium will actually be played. One way to solve these games is again the Schelling point (if the management has given in a few times before, it is reasonable they will do it again).

3.3.5 Ultimatum game

This game has been practically studied by many game theorists, economists and psychologists. It is usually stated like this:

Player A is given a sum of money. He has to offer Player B a portion of this money. If Player B accepts his proposal, they split the money according to the proposal. But if Player B rejects the offer, both of them get nothing. Therefore Player A makes an **ultimatum**.

The classic game theoretic solution is that Player B should and will accept any offer, as any monetary gain is preferred to none. So Player A should make the minimal offer possible and this offer will be accepted. This game has been modified and practically played in many ways, but most of the time, the game theoretic solution was in reality not used, as minimal offers were rejected by Player B. There are various explanations for that using the knowledge of psychology, sociology or the concepts of evolutionary game theory.

(BEARDEN, 2001)

3.3.6 Dictator game

The dictator game is a variation of the ultimatum game. The game goes as follows:

Player A is given a sum of money and he has to propose a split with player B. Player B is then given his portion and player A keeps the rest.

This is not exactly an example of a game, but it is a case of strategic decision of one player (it is a degenerate game). The game theoretic solution in this case is to offer Player B nothing and keep all the money. But in the experiments, again this solution was not used. Majority of the results showed that Player A proposed a non-zero sum of money to Player B, displaying **altruism**. It proves that players in real life consider not only monetary gains.

(BEARDEN, 2001)

3.4 Dynamic games

In dynamic games the players do not make their decisions at the same time, but they do it in a given sequence. The order of moves affects the solutions for these games,

because the players moving later in the game observe the moves made before and make their decisions considering that.

3.4.1 Extensive form

To completely describe a dynamic game, we can use its **extensive form**. The extensive form consists of:

1. Set of players – a list of players playing the game. One special player is Nature – it represents exogenous actions. Nature does not play a role in every game. Nature's moves are usually associated with external events that happen with some probability.
2. Order of events – this order is usually described by the **game tree**. It describes the possibilities each player has at certain points in the game. The game tree has to have a unique initial node, the **root**, from which the game begins. A finite path of predecessors from each node of the tree to the root has to exist. Each node is immediately preceded by only one node (except from the root). Each path of the game has to reach a **terminal node**. The path to the terminal node represents the individual decisions that had to be made to get to that terminal node – the **game history**.
3. Order of moves – each node has a player assigned to them. Nature, if present, moves first and determines the outcome.
4. Available actions – at each node the player whose order it is to play has a set of available actions. The number of actions is equal to the number of nodes that immediately success the current node. Each successor is associated with one unique action.
5. Information sets – Information sets are used to model games of imperfect information. When certain nodes are grouped into information sets, the player cannot distinguish at which node he really is – he does not know what the preceding decisions were. The available actions have to be the same for all of the nodes in one information set (otherwise the player can actually distinguish the nodes).
6. Payoffs – each terminal node is associated with a certain payoff for each player. Nature does not have payoffs.

(VEGA-REDONDO, 2003)

I will explain the necessary tools needed to solve dynamic games on the following example:

Two influential political parties are arguing about two new laws that are going to be voted in the parliament. The law that is going to be voted for the first is the new Commercial code, which is going to change the way the minimum wage is calculated – effectively it is going to get higher. The Labour party is a supporter of this law. The second law legalizes same-sex marriage. The Liberal party is a supporter of this law. Liberal party does not really care about the Commercial code law, and the Labour party does not care about the same-sex marriages law. But it is of value if they vote the same (to show the public that they stand together). Neither of the parties alone has the power to push the law they want, but together they have enough members to push the vote. It is known that the Labour party is going to vote for the Commercial code and the Liberal party for the same-sex marriages. The Labour party will know if the Liberal party voted for or against the Commercial code before the same-sex marriages law is going to be voted. Both parties know before what is the stance of the other party on these laws – it is a game of perfect information.

3.4.2 Game tree

The resulting sequential game can be illustrated using a special kind of graph - **a game tree**:

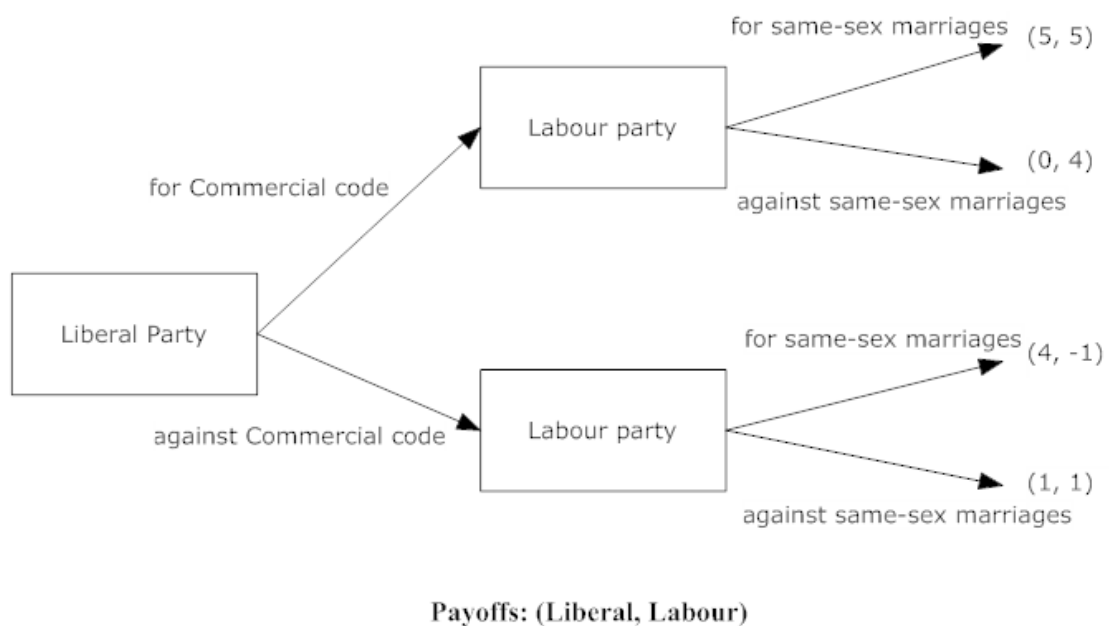


Figure 3.1 Game tree for the politics game

The nodes in the tree represent the points where the parties have to make their decisions. Both players know the previous decisions. The outcomes of the game can be also represented using a table:

- CC+/CC- means for/against the Commercial code
- SSM+/SSM- means for/against the same-sex marriages

		Liberal	
		CC+	CC-
Labour	SSM+	(5, 5)	(4, -1)
	SSM-	(0, 4)	(1, 1)
Payoffs: (Liberal, Labour)			

Table 3.11 Payoffs for the politics game

First we can find the Nash equilibria in this game using the strategic form of this game. The strategic form shows the payoffs for each strategy profile:

		Liberal	
		CC+	CC-
Labour	(SSM+, SSM+)	(5, 5)	(4, -1)
	(SSM+, SSM-)	(5, 5)	(1, 1)
	(SSM-, SSM+)	(0, 4)	(4, -1)
	(SSM-, SSM-)	(0, 4)	(1, 1)
Payoffs: (Liberal, Labour)			

Table 3.12 Strategic form of the politics game

From the strategic form of the game we can try to find the Nash equilibria by evaluating each strategy profile.

- If the Labour party plays (SSM+, SSM+) strategy, then Liberal party will choose to play CC+. If the Liberal plays CC+, then the Labor party cannot do better than play (SSM+, SSM+), so {CC+, (SSM+, SSM+)} is a Nash equilibrium.

- If the Labour party plays (SSM+, SSM-), then Liberals are better off playing CC+. If the Liberal party plays CC+, then the Labor party cannot do better than play (SSM+, SSM-), so {CC+, (SSM+, SSM-)} is a Nash equilibrium.
- If the Labour party plays (SSM-, SSM+), then Liberals are better off playing CC-. If the Liberal party plays CC-, then playing (SSM+, SSM+) is better than playing (SSM-, SSM+), so it is not a Nash equilibrium.
- If the Labour party plays (SSM-, SSM-), then Liberals are better off playing CC-. If the Liberal party plays CC-, then Labour party cannot do better than play (SSM-, SSM-), so {CC-, (SSM-, SSM-)} is a Nash equilibrium.

3.4.3 Backwards induction

We have found 3 Nash equilibria for this game. To find out which one is the most likely to occur we can use **backwards induction** (without the player Nature).

“This procedure has six steps:

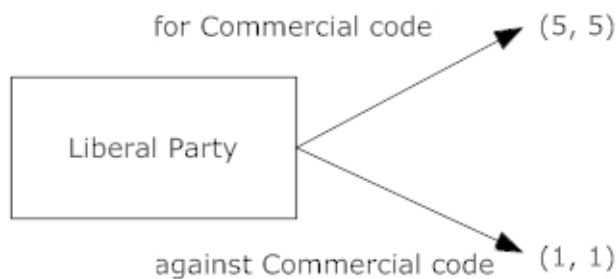
1. Start at the terminal nodes of the game and trace each one to its immediate predecessor, which will be a decision node for some player. These decision nodes are either “trivial”, “basic”, or “complex”. A decision node is **basic** if each of its branches leads to exactly one terminal node. A basic node with only one branch is **trivial**. A decision node is **complex** if it is not basic. If you reach a trivial decision node, then keep moving up the tree until you reach either a complex or nontrivial basic decision node or until you can go no further.
2. Find the optimal move at each basic decision node reached in step 1 by comparing the payoffs the player obtains at each terminal node reached from this decision node. Notice that every path between a basic decision node A and a terminal node B starts at a unique branch of A. The branch that leads to the highest payoff for the player is the optimal move to make at that node.
3. Erase all the non-optimal branches that originate from each of the basic decision nodes you examined in step 2. Each of these basic decision nodes becomes trivial.
4. You now have a new game tree that is simpler than the original one. If in step 1 you arrived at the root of the tree, you are now done.

5. If you have not yet reached the root, then go back to step 1 and start all over again. In this way, you work your way step by step toward the root.
6. For each player, collect together the optimal decisions at each of the player's decision nodes. This collection of decisions constitutes that player's optimal strategy in the game."

(BIERMAN and FERNANDEZ, 1998, p.129-130)

Now we can apply this rule to the previous example.

First we find the optimal decision of the Labour party at the bottom decision node. If they support SSM, they get a payoff of -1, if they do not support, they get a payoff of +1, so the optimal decision is to not support SSM. In the upper node the optimal solution is to support SSM and get a payoff of 5 compared to 4 when voting against SSM. After eliminating the non-optimal branches the game tree looks as depicted in figure 3.3.



Payoffs: (Liberal, Labour)

Figure 3.2 Game tree after elimination of non-optimal branches

The only thing left is to find the optimal decision for the Liberal party at the remaining node. The optimal decision is to support the CC and get a payoff of 5 compared to not supporting and payoff of 1. So the only Nash equilibrium strategy profile left after the backwards induction is applied is {CC+, (SSM+, SSM-)}.

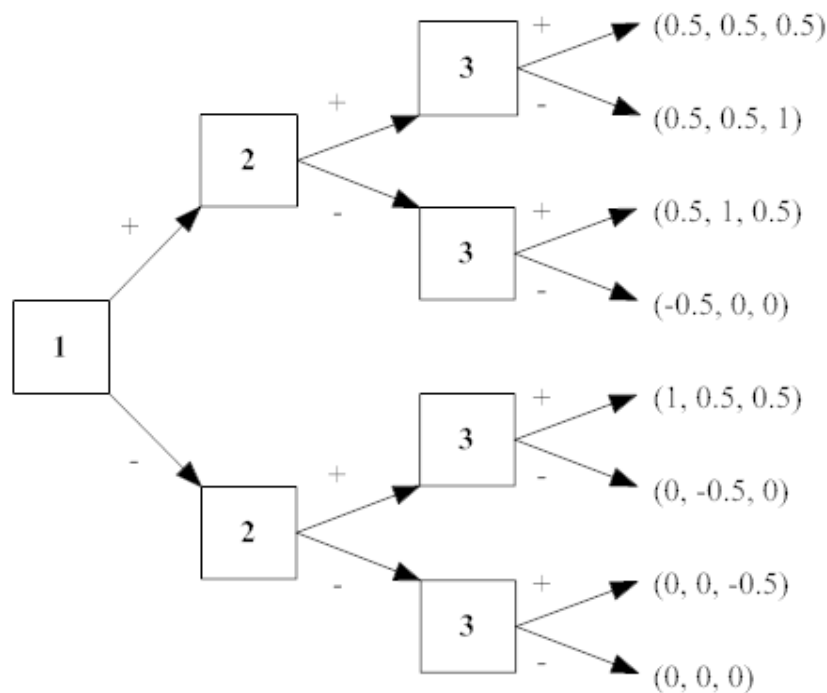
3.4.4 Threats, promises and credibility

It is possible that the Labour party will try to force the Liberal party into voting for the CC by making a **threat**: „We will not vote for SSM if you do not vote for the CC“. From the previous solution we can see that if the Liberal party votes for against the CC,

it is in Labour party's best interest to vote against SSM. It means that the threat made by the Labour party is **credible** and the Labour party should not ignore it. They can also make a **promise**: „We will vote for SSM if you vote for the CC“. Again it is in their best interest to keep their promise, so this promise is credible. Nash equilibria can also contain incredible threats - threats, which are not rational for the issuer to realize.

I will illustrate the idea of an incredible threat on the following example:

Three top managers (we will call them 1, 2, 3) in a company are voting whether to cut their working hours by one hour. The company policy is that the employees of the company are informed not only about the result of the voting, but also about who voted for and who voted against the proposition. The manager that votes for the proposition is going to anger his subordinates and they will in response stall him at work for half an hour. The managers vote one after another and the vote is public. The game tree for this game looks is shown in the following figure.



Payoffs: (1, 2, 3)

Figure 3.3 Game tree for the managers game

Using backwards induction we can find that the equilibrium solution will be the strategy profile $\{-, +, +\}$. Say for example that manager 3 would threaten manager 1 with saying: “If you do not vote for the proposition, I will not vote for it either.” We can see that this threat is not credible because it is in his best interest to vote for the proposition to get a payoff of 0.5 compared to payoff of 0 if he would have kept his word.

3.4.5 Subgame perfect equilibrium

To find Nash equilibria that do not involve incredible threats we can use the concept of **subgame perfect equilibrium**. Definition of a **subgame** follows:

“The subgame G_S of the game G_T is a game constructed as follows:

1. G_S has the same players as G_T although some of these players may not make any moves in G_S
2. The initial node of G_S is a subroot of G_T and the game tree of G_S consists of this subroot, all its successor nodes, and the branches between them.
3. Each player’s payoffs at the terminal nodes of G_S are identical to the payoffs in G_T at the same terminal nodes.”

(BIERMAN and FERNANDEZ, 1998, p.133-134)

Every game is a trivial subgame of itself. A **proper subgame** is a nontrivial subgame of a game.

Subgame perfect equilibrium:

The strategy profile selected by the backward induction is the subgame perfect equilibrium of a dynamic game with perfect information. This is also the equilibrium for all proper subgames of G .

(BIERMAN and FERNANDEZ, 1998, p.133-134)

4 Proposals of solutions and their contributions

4.1 Oligopoly models

In this chapter four examples of the oligopoly models (games) will be presented:

- The Cournot game
- The Stackelberg game
- The Bertrand game
- The Contestable monopoly model

Some of these models were created before the studies of game theory. In the following subchapter we will show how they are used together with game theory to solve simplified real world examples.

4.1.1 Cournot model

Introduced in 1838 by French philosopher, economist and mathematician Antoine Augustin Cournot this model (or game) can be used to describe various industry structures. In this model all of the competing companies on the market compete by choosing their output and try to maximize their profits. All of them decide on the output independently at the same time and all of them sell for the same market price. The market price of the goods in the Cournot model is a function of the total amount of goods, denoted by Q_T , produced by all of the companies together and it equals the market clearing price P :

$$P = P(Q_T) \text{ and } \frac{dP(Q_T)}{dQ_T} < 0$$

, where

$$Q_T = \sum_{i=1}^n Q_i$$

and Q_i is the individual output of firm i .

The model requires the following assumptions:

- Price is not a strategic variable
- Products of the companies are identical
- Products appear at the market simultaneously

- Firms do not cooperate
- The number of firms is fixed
- The market price decreases when the total output rises

(FUDENBERG and TIROLE, 1998)

The following example illustrates the use of this model.

Two firms form a duopoly on the gold mining market: We Mine and Shiny Gold. Both of them sell the gold that they mine each month at a gold market. The quantity of gold mined by We Mine will be denoted Q_{WM} and Shiny Gold's quantity Q_{SG} . The market price at the gold market is given by the following demand equation in €ounce of 24-karat gold:

$$P = \begin{cases} 840 - 4 * Q_T & \text{if } 0 \leq Q_T \leq 210 \\ 0 & \text{if } Q_T > 210 \end{cases}$$

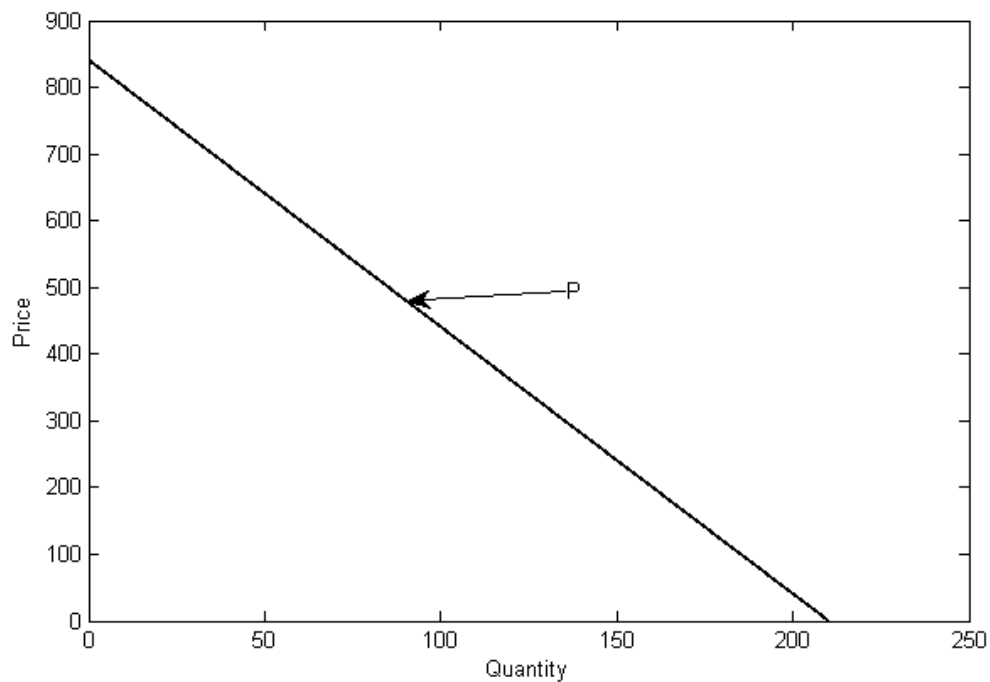


Figure 4.1 Demand curve for the gold market

Each month both firms have to decide how much gold they want to mine – they have to select a strategy. They decide at the same time and independently. The quantities are given in thousands of ounces and revenue, cost and profit in thousands of euro.

The mining and transporting costs of the companies are identical and are 600€ounce.
Total costs for each company are:

$$TC_i = 600 * Q_i \quad \text{for } i = \text{We Mine, Shiny Gold}$$

Now, we can write the profit function for each of the gold mining companies.

Profit function for the We Mine company:

$$\pi_i(Q_{WM}) = (840 - 4 * (Q_{WM} + Q_{SG})) * Q_{WM} - Q_{WM} * 600$$

Profit function for the Shiny Gold company:

$$\pi_i(Q_{SG}) = (840 - 4 * (Q_{WM} + Q_{SG})) * Q_{SG} - Q_{SG} * 600$$

The profits are considered the payoffs in this game. The companies play a static game with complete information (the profit functions are common knowledge).

4.1.1.1 Monopoly

To compare solutions to this game we will first assume that there is only one company - We Mine. In this case its profit function will look like this:

$$\pi_{WM}(Q_{WM}) = (840 - 4 * Q_{WM}) * Q_{WM} - Q_{WM} * 600$$

After simplification:

$$\pi_{WM}(Q_{WM}) = 240 * Q_{WM} - 6 * Q_{WM}^2$$

To find the optimal output, we now only have to find the maximum of the profit function using simple calculus. To find the maximum we have to put the derivative of the profit function equal to 0:

$$240 - 12 * Q_{WM} = 0$$

And the output associated with the maximum profit is:

$$Q_{WM} = 20$$

Profit at this output level is equal to:

$$\pi_{WM}(20) = 240 * 20 - 6 * 20^2 = 2400$$

So in the case of one company on the market, this company would produce 20 000 ounces of gold each month and make a profit of 2 400 000€

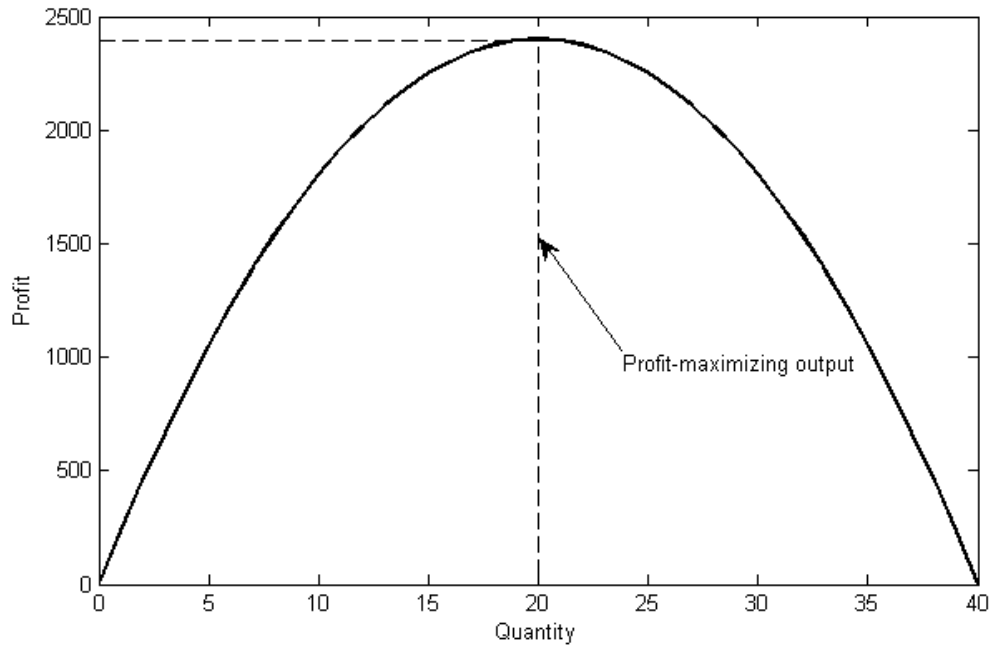


Figure 4.2 Graph of the profit function for firm We Mine

4.1.1.2 Duopoly

We can return to the problem with two competing companies. To illustrate the problem in a better fashion, we can use the **isoprofit** curves. Isoprofit curves show all of the different levels of outputs of both mining companies where one of them makes a certain level of profit. For a given profit of π_x for the company We Mine, the isoprofit curve for the Shiny Gold company is the following function:

$$Q_{SG} = 60 - \frac{\pi_x}{4 * Q_{WM}} - Q_{WM}$$

The following graph illustrates the isoprofit curves for the company We Mine for these levels of profit 0, 250, 500, 1000 and 2000.

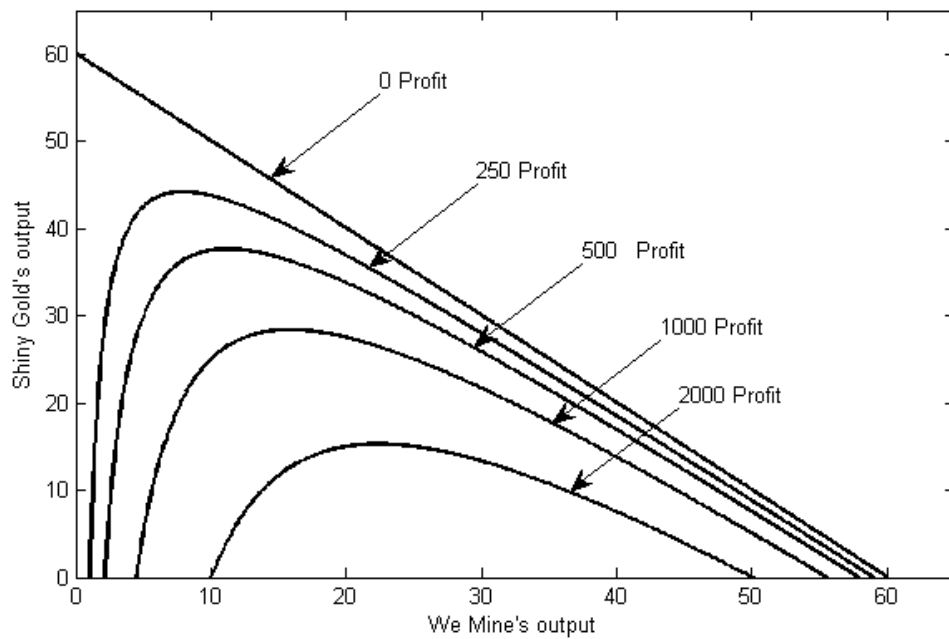


Figure 4.3 Isoprofit curves for We Mine

Now we have to find the **best responses** for the level of output selected by the competitor. For example if Shiny Gold selects the output of 0 units, then the best response for the We Mine company will be to select the output of 20 as if it was alone on the market (actually they select their output at the same time, so the best response output will be selected next month). For each non-zero output level we have to find an isoprofit curve that is tangent to the horizontal line of the output at the height of Q_{SG} . For example if the Shiny Gold's output is 30, then the best response for the We Mine will be to select the output of 15. It is illustrated in Figure 4.4.

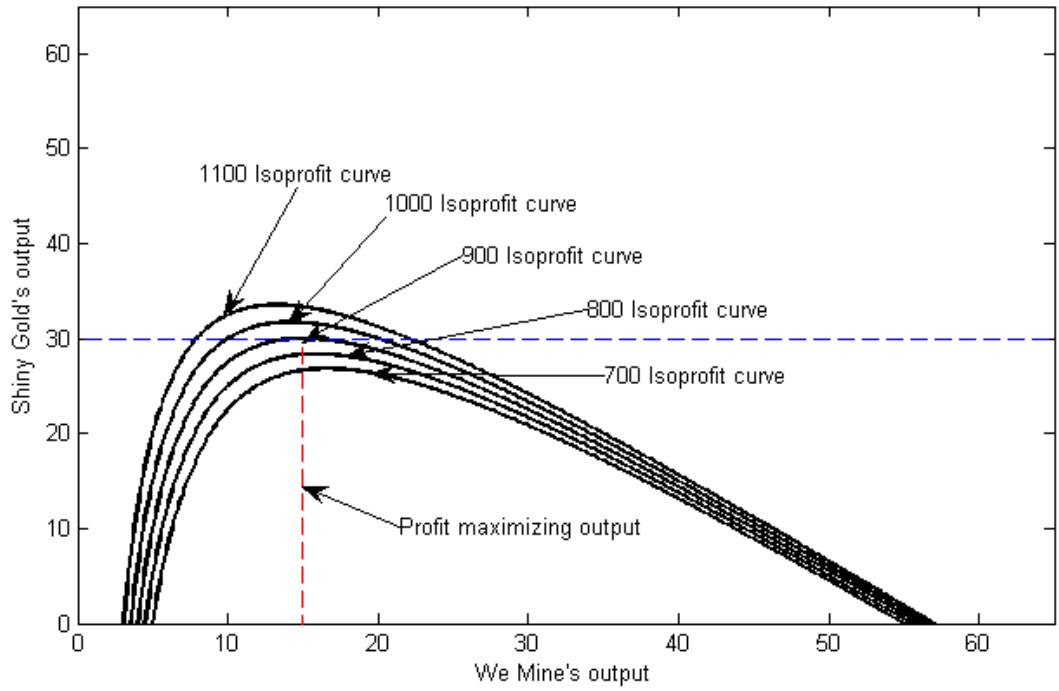


Figure 4.4 Best response for the Shiny Gold's output of 30

To find the best response for each one of the levels of output we have to find the **best response** functions for both companies, that is, the functions that assign a level of output to each level of competitor's output in a way to maximize profit. We can calculate this function for the We Mine by taking the partial derivative of their profit function with respect to Q_{WM} and calculating the level of output when it equals 0.

$$\frac{\partial \pi_{WM}(Q_{WM}, Q_{SG})}{\partial Q_{WM}} = 240 - 8 * Q_{WM} - 4 * Q_{SG} = 0$$

Solving for Q_{WM} , the best response function for We Mine is:

$$(4.1) \quad Q_{WM}^{BR}(Q_{WM}, Q_{SG}) = 30 - 0.5 * Q_{SG}$$

Shiny Gold's best response function:

$$(4.2) \quad Q_{SG}^{BR}(Q_{WM}, Q_{SG}) = 30 - 0.5 * Q_{WM}$$

To find the quantities that the companies will choose according to Cournot model, we now just have to find the intersection of these two best response functions. We can depict it graphically as shown in Figure 4.5.

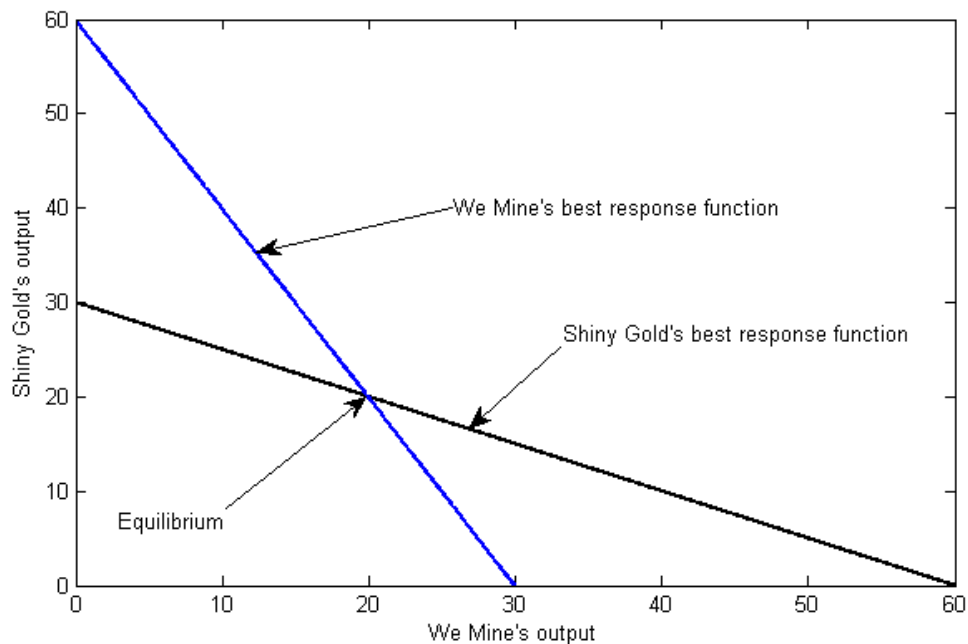


Figure 4.5 Intersection of the best response functions

We can also find the equilibrium mathematically by solving a system of linear equations 4.1 and 4.2. The solution is that both the firms will mine 20 units (20 000 ounces), sell them for a price of 680€/ounce and earn a profit of 1600 (1 600 000€). This is the **Cournot duopoly equilibrium** for this game. This is also the Nash equilibrium of this game as both players maximize their profits when they are playing their best responses and they are both playing their best responses in this point. We can also receive the same result by iteratively eliminating dominated strategies.

But Cournot model can be used to model more than monopoly and duopoly market a structure. By increasing the number of companies and decreasing their market share we are also able to model perfect competition.

4.1.1.3 Perfect competition

We can modify the previous gold mining example by adding more firms to the market. Now for N gold mining companies the profit function for the i -th company will be given by.

$$(4.3) \quad \pi_i(Q_1, \dots, Q_N) = \begin{cases} (840 - 4 * Q_T) * Q_i - 600 * Q_i & \text{if } Q_T < 210 \\ -600 * Q_i & \text{if } Q_T \geq 210 \end{cases}$$

When

$$Q_T = Q_i + Q_{-i}$$

we can rewrite 4.3 like this:

$$(4.4) \quad \pi_i(Q_1, \dots, Q_N) = \begin{cases} (240 - 4 * (Q_i + Q_{-i})) * Q_i & \text{if } Q_i < 60 - Q_{-i} \\ -600 * Q_i & \text{if } Q_i \geq 60 - Q_{-i} \end{cases}$$

From 4.4 we can see that the profit is negative when the number output of companies reaches 60. From that we know that if the output of the industry is ≥ 60 it does not make sense to mine and lose money (the costs are not 0).

So the best response function for the i -th company will be not to mine if the industry output is ≥ 60 . To find the best response when the output is < 60 we have to take a partial derivative of the profit function with respect to Q_i and put it equal to 0 to find the maximum.

$$\frac{\partial \pi_{WM}(Q_i, Q_{-i})}{\partial Q_i} = 240 - 8 * Q_i - 4 * Q_{-i} = 0$$

And the best response is:

$$(4.5) \quad Q_i^{BR}(Q_1, \dots, Q_N) = 30 - \frac{Q_{-i}}{2} \quad \text{if } Q_{-i} < 60$$

All of the companies are identical, so they have the same response function 4.5, which means they have the same output-the best response output. That means that the strategy profile $\{Q_1^*, \dots, Q_N^*\}$ is the Nash equilibrium, where each company's output is equal to the best response output- $Q_i^{BR}(Q_1, \dots, Q_N)$ or Q^* . So the following must be true for the equilibrium output Q_{i-1}^* :

$$Q_{i-1}^* = (N - 1) * Q^*$$

If we substitute this into the equation 4.5 we obtain:

$$Q^* = 30 - \frac{(N - 1) * Q^*}{2}$$

After simplification, the equilibrium output of one company in perfect competitive industry is:

$$Q^* = \frac{60}{1 + N}$$

The output of N firms (the whole industry) is:

$$\frac{60 * N}{1 + N}$$

For $N \rightarrow \infty$ the output of the industry equals 60 000 ounces of gold with a profit of 0.

The problem of this model is that in real world the companies usually do not compete by setting the quantities that they produce and they set their own prices.

4.1.2 Stackelberg model

This model (or game) was introduced by Heinrich Freiherr von Stackelberg, a German economist, in 1934. It models a duopoly in which there is a company that is a leader and another one that is a follower. The game is sequential; the leader moves first and sets the quantity he is going to produce. Then the follower observes this and sets his quantity. Other important assumptions are:

- Both players are perfectly informed about the game structure and payoffs
- The leader knows that the follower will follow him

We will demonstrate the usage of this model on the following example:

There is a monopolist, Metalheads T's (MT), producing T-shirts for a certain niche market. Each month he has to decide how many T-shirts he wants to order from the manufacturer in China. A new company, Brutal T-shirts (BT), wants to enter this market. They can observe how many T-shirts the incumbent company is going to produce and after that they have to decide how many T-shirts they will produce. The monopolist knows that this competitor will make its decision based on their decision and they have to set their quantity while keeping that in mind. For the sake of simplicity the cost structure is the same for both companies. The costs associated with making one T-shirt are 1€ The demand for these T-shirts for the specific market is given by this demand curve equation:

$$(4.6) \quad P = 801 - 2 * Q$$

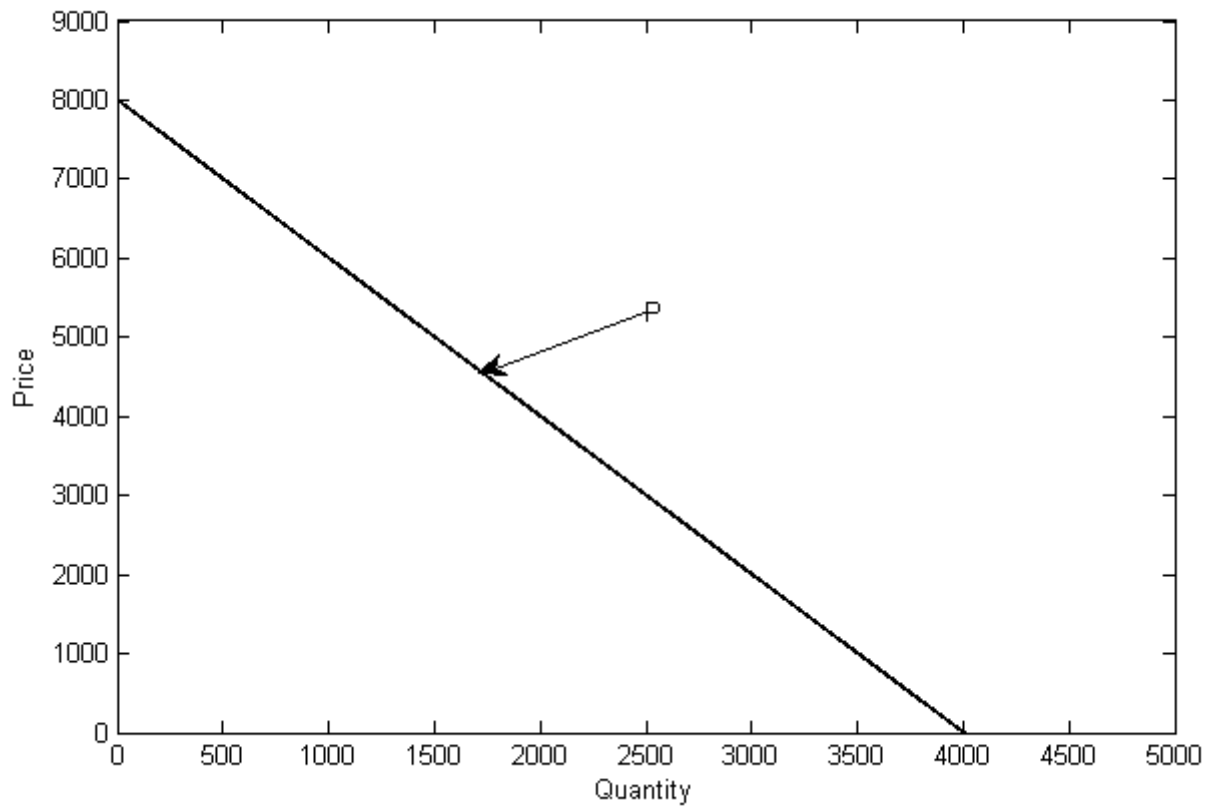


Figure 4.6 Demand curve for the T-shirts market

4.1.2.1 Monopoly

The following solution is for the case that the monopolist is alone on this market. How many T-shirts will he produce? He will produce the amount of T-shirts that gives him the highest profit. His profit function is:

$$\pi(P, Q) = (P - 1) * Q$$

After substituting in the demand curve equation 4.6:

$$\pi(Q) = (801 - 2 * Q - 1) * Q$$

After simplification:

$$\pi(Q) = 800 * Q - 2 * Q^2$$

To find the maximum we have to take the derivative with respect to Q and put it equal to 0:

$$\frac{\partial \pi(Q)}{\partial Q} = 800 - 4 * Q = 0$$

$$Q^* = 200$$

The optimum production for the monopolist is 200 T-shirts, which earns him a profit of:

$$\pi(200) = 800 * 200 - 2 * 200^2 = 80\,000\text{€}$$

So if the monopolist was on the market alone, he would produce 200 T-shirts, sell each for the price of 400€ and make a profit of 80 000€

4.1.2.2 Duopoly

Now, the monopolist has to consider that after he makes his decision, he is going to be followed by the competitor. The resulting game is a sequential (dynamic) game in which the leader moves first and the follower moves second. The production of MT is Q_{MT} and the BT is Q_{BT} , $Q = Q_{MT} + Q_{BT}$. The demand equation now changes to:

$$P = 801 - 2 * (Q_{MT} + Q_{BT})$$

The profit equation of the monopolist now depends on both Q_{MT} and Q_{BT} . The profit equation looks like this:

$$\pi_{MT} = (801 - 2 * (Q_{MT} + Q_{BT}) - 1) * Q_{MT}$$

After simplification:

$$(4.7) \quad \pi_{MT} = 800 * Q_{MT} - 2 * Q_{MT}^2 - 2 * Q_{MT} * Q_{BT}$$

The profit equation of BT is:

$$\pi_{BT} = 800 * Q_{BT} - 2 * Q_{BT}^2 - 2 * Q_{MT} * Q_{BT}$$

To solve this game we have to reason backwards. First we have to find out how will BT react to MT's output choice. To find the output which maximizes profit of BT we have to take the partial derivative of their profit function with respect to Q_{BT} and put it equal to 0. The Q_{MT} is considered a constant at this point – the monopolist has already set the quantity.

$$\frac{\partial \pi_{BT}(Q_{MT}, Q_{BT})}{\partial Q_{BT}} = 800 - 4 * Q_{BT} - 2 * Q_{MT} = 0$$

$$(4.8) \quad Q_{BT}^{BR} = 200 - \frac{Q_{MT}}{2}$$

The equation 4.8 gives us the answer to the question: How many T-shirts is BT going to make given that MT makes Q_{MT} T-shirts? It is their best response function. We assume the perfect knowledge of the game, so the leader - MT knows this function. His profit

depends on the choice of BT's output so we can substitute the BT's response function into MT's profit function 4.7.

$$\pi_{MT} = 800 * Q_{MT} - 2 * Q_{MT}^2 - 2 * Q_{MT} * (200 - \frac{Q_{MT}}{2})$$

After simplification:

$$(4.9) \quad \pi_{MT} = 400 * Q_{MT} - Q_{MT}^2$$

To find the level of output which gives MT the maximum profit, we just have to find the maximum of the 4.9 function.

$$\frac{\partial \pi_{MT}(Q_{MT})}{\partial Q_{MT}} = 400 - 2 * Q_{MT} = 0$$

$$Q_{MT} = 200$$

MT produces 2000 T-shirts and makes a profit of:

$$\pi_{MT} = 400 * 200 - 200^2 = 40\,000\text{€}$$

The price for one T-shirt in this situation is 200€

When we compare the profit of the monopolist when he is alone and when he has a competitor, we see that his profit was reduced to half.

BT will make:

$$Q_{BT} = 200 - \frac{200}{2} = 100$$

T-shirts and make a profit equal to half of MT's profit (half of the number of T-shirts times the same price), which is 20 000€. None of the players can do any better by changing their output -> it is a Nash equilibrium.

This model shows that the entry of a new competitor reduces prices and also profits for the incumbent firm.

This model also has another important aspect - the company that moves first always makes twice the money of the company that moves last. The first company has the **first mover advantage**. This is not generally true for every game, but it is a property of a portion of them.

4.1.3 Bertrand model

This model (or game) was introduced by Joseph Louis François Bertrand, a French mathematician. The difference between Bertrand and Stackelberg or Cournot model is

that the companies in Bertrand model compete with prices instead of quantities. The Bertrand game is a static game with perfect information. The companies make their decisions independently and simultaneously. Other assumptions:

- Product is homogeneous.
- Customers are rational – they buy for the lowest price

To illustrate the usage of this model, consider the following example:

There are two chemical factories in city that form a duopoly on the market with sulfuric acid - Acidity (A) and Bang (B). We assume that both of them produce sulfuric acid with constant marginal costs of 5€ per liter of acid and do not have any fixed costs. Companies make their prices known independently and at the same time. The product is exactly the same from both companies, so the customer only cares about the pricing. If the prices are the same, the companies split the market. There is a positive demand for the sulfuric acid at a price of 5€ per liter.

There is no point in selling under the marginal costs of producing the acid so we do not have to consider prices below the 5€ level. If company A sets its prices at for example 6€ then it is reasonable for the management of B to set the price slightly lower than 6€, capture the whole market and make profit. If they choose a price higher than A, then they will have zero market share and make zero profit. If they choose the same price as A, then they will only capture half of the market and make lower profit than they would have made if they had chosen a lower price than A. But management of A has to consider the same. This concludes that both companies will charge the price of 6€, which is equal to their marginal costs and is the Nash equilibrium as neither of the companies can do any better.

To allow the companies to make profit in the Bertrand model we have to change our assumptions about the game. One of the possible modifications is to limit the capacities of the competitors.

4.1.4 Contestable markets

This is the newest of the presented models (or games) introduced by William Jack Baumol, American economist, in 1982. A contestable market is a market where only a

small number of firms operate, but they cannot make excessive profits as they are under the threat of entry by potential competitors.

“A **contestable market** is one for which:

1. An unlimited number of potential firms exist that can produce the (homogeneous) with a common technology.
2. Entry in to the market does not involve a sunk cost, that is, an expenditure that cannot be completely recovered should the firm decide to leave the market.
3. Firms are price-setting Bertrand competitors.

A **contestable monopoly** is a contestable market with a Nash equilibrium in which exactly one firm supplies the entire market. In a contestable monopoly, since the threat of entry drive monopoly rents to zero, the firm will produce the greatest output at which it remains financially viable. This is referred to as a second-best output.”

(BIERMAN and FERNANDEZ, 1998, p.52-53)

An example for this kind of market can be a market with taxi services in a small town. Consider two taxi companies: AB Taxi (AB) and Okey Taxi (OK). To provide the service, the companies only need to have a number of cars and drivers (we will assume that there is no need for a special license). There are fixed costs associated with buying or leasing the cars needed, but they are not sunk costs as the cars can be sold or used in another city. Q_{AB} , P_{AB} , Q_{OK} and P_{OK} are the respective quantities of transported passengers and prices for one ride for both companies. The number of clients is in hundreds per month and prices are in €per ride. The companies have the same costs, which are:

$$C = \begin{cases} 4 + 4 * Q & \text{if } Q > 0 \\ 0 & \text{if } Q = 0 \end{cases}$$

If one of the companies decides to quit the market, they can sell their cars and that way avoid any losses (no sunk costs). Both companies announce their prices at the same time. The demand curve is given by the equation:

$$P = 15 - 2.5 * Q$$

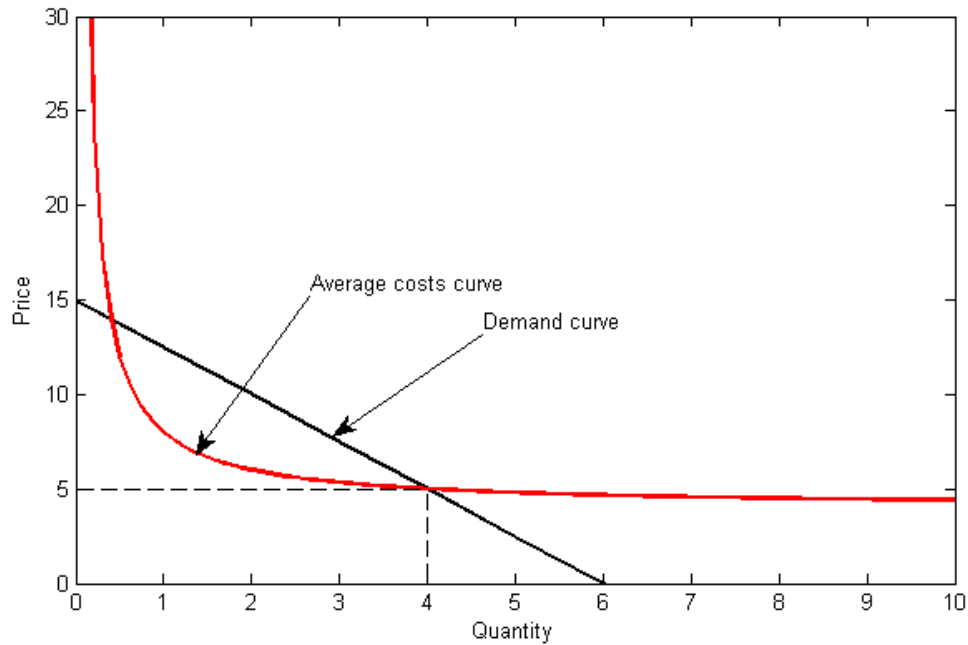


Figure 4.7 Demand and average costs curves for the contestable monopoly

From the picture we can see that at the price of 5€ per ride there is a demand of 400 customers. Now, we will show that there are two Nash equilibrium strategies in this game for both companies (the companies are the same and so the strategies are symmetrical):

- AB sets its price to 5€ and transports 400 passengers and OK transports no one.
- OK sets its price to 5€ and transports 400 passengers and AB transports no one.

It is enough to show that the first strategy is a Nash equilibrium and that neither of the players can do any better than playing that strategy. If OK sets its price lower than 5€ they will catch the whole market but they will lose money on it, unless they refuse to transport anyone, as the average costs are equal to 5€. If OK sets its price to 5€ exactly, they will transport half of the passengers and also lose money, unless they refuse to transport anyone. If they set their price higher than 5€, they will not transport anyone as all the passengers will go to the competition. They will not make any profit, but if they sell their cars to another company (or use them in a different city) they will also not lose any money. So it is the optimal strategy for OK to not transport anyone and sell its cars. It is also optimal for AB to set its price to 5€, as the lower price would make them

lose money on transporting and higher price would make room for a competitor to enter the market and make positive profits.

The important property of this model is that the **possibility of entry** (not the actual entry) is in some special markets enough to keep the prices low enough to discourage potential competitors from entering the market.

4.1.5 Summary of the oligopoly models

Both Stackelberg and Cournot models model companies competing by setting the quantities they want to produce, while Bertrand model models them competing on price. Each model has its uses depending on the structure of the market whose behavior we try to predict. All of these models in the form given here are too simple to model real life markets. The important assumptions to consider are the one-time nature of the price setting and also the homogeneity of the companies' products in these games which is in most cases not true in real markets.

4.2 Economic views comparison

In this chapter different ways of reasoning will be presented on an example of a strategic investment.

There are two companies on a pharmaceutical market – Friendly Pharm (FP) and Medicinal Goodness (MG). Both of them produce an analgesic medicine. These two products are perfect substitutes. Quantities are in kilograms, revenues, costs and profit are in thousands of euro and prices are in €/kg. These companies both have the same marginal costs, $MC_1=MC_2=MC$ equal to:

$$MC = 25$$

It means that it costs them 25€ to produce one kilogram. The demand curve for these products is:

$$(4.10) \quad P = 109 - \frac{1}{6} * Q$$

Q is the sum of quantities that the companies produce:

$$Q = Q_{FP} + Q_{MG}$$

They produce the same quantity of these products, $Q_{FP}=Q_{MG}=168$, which is the Cournot equilibrium output. At this output the price of one kilogram is:

$$P = 109 - 56 = 53\text{€}$$

They both make a yearly profit of:

$$\pi = (53 - 25) * 168 = 4704$$

It means that the profit of each company equals 4 704 000€

FP is considering an investment in a new manufacturing process in their factory. This investment would lower their marginal costs to:

$$MC_{FP} = 20$$

The cost of this new process is 1 000 000€

Now we will compare three different perspectives on this investment:

- “Accountant’s” perspective
- “Economist’s” perspective
- “Game theorist’s” perspective

4.2.1 Accountant’s point of view

From this perspective we will count the money that this process will save us and compare it with the cost of the machine.

We produce 168 kilograms with costs 5€ lower than before, so we save:

$$168 * 5 = 840$$

The savings are 840 000€ in variable costs, but the cost of the new process is 1 000 000€. We can see that the costs of the new process are much higher than the savings it brings. So, in conclusion, we advise not to invest in this process as it will bring no advantage.

4.2.2 Economist’s point of view

From this perspective we can see that when production costs are lowered by the new process, we not only save money, but we can also produce more. Our residual demand curve is:

$$(4.11) \quad P_R = 109 - \frac{1}{6} * (Q_{FP} + 168) = 81 - \frac{1}{6} * Q_{FP}$$

Quantity which maximizes profit is in the point where marginal revenue equals marginal costs:

$$(4.12) \quad MR_{FP} = MC_{FP}$$

We already know the marginal costs. We can calculate total revenue, denoted by TR, by multiplying 4.11 with the produced quantity Q_{FP} :

$$(4.13) \quad TR_{FP} = P_R * Q_{FP} = \left(81 - \frac{1}{6} * Q_{FP}\right) * Q_{FP} = 81 * Q_{FP} - \frac{1}{6} * Q_{FP}^2$$

From 4.13 we can calculate the marginal revenue by taking its derivative:

$$MR_{FP} = \frac{\partial TR_{FP}(Q_{FP})}{\partial Q_{FP}} = 81 - \frac{1}{3} * Q_{FP}$$

Now we substitute it into 4.12:

$$81 - \frac{1}{3} * Q_1 = 20$$

And we obtain Q_{FP} :

$$Q_{FP} = 183$$

We can also solve it graphically:

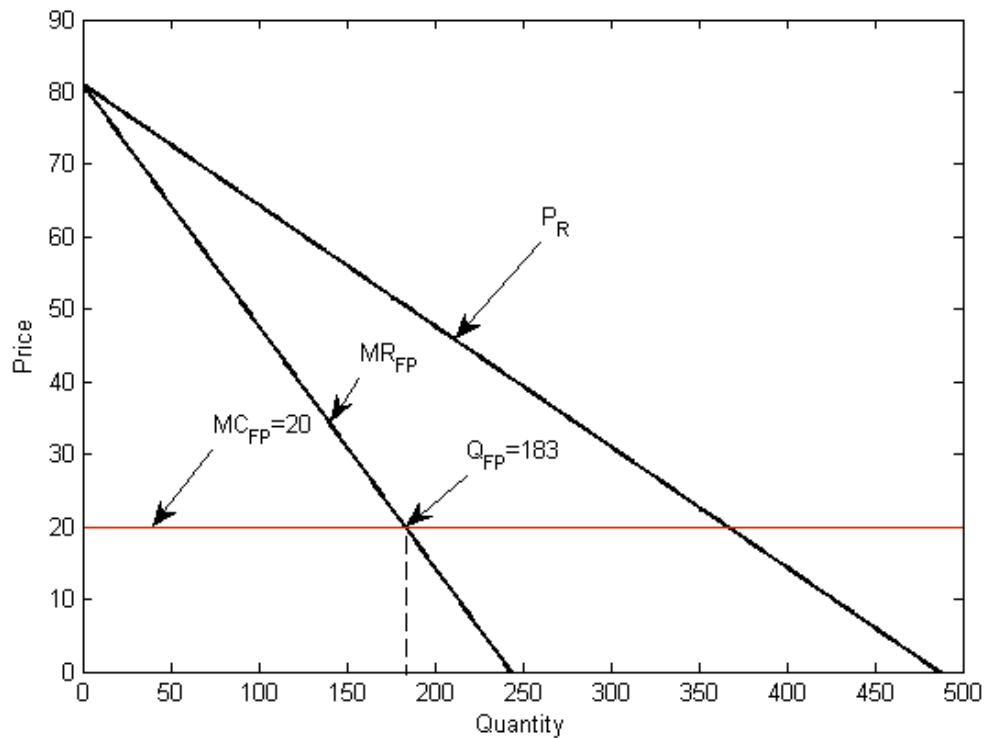


Figure 4.8 Graphical solution – „economist’s“

With the output of 183 kilograms the new price is:

$$P = 109 - \frac{1}{6} (183 + 168) = 50.5\text{€}$$

And the profit is:

$$\pi_{FP} = (50.5 - 20) * 183 = 5581.5$$

The new profit is 5 581 500€ The difference between the profit before adopting the process and after adopting is 877 500€ which is still lower than the costs of the new process. So, in conclusion, we again advise not to invest in this process as it will bring no advantage.

4.2.3 Game theorist's point of view

The previous view did not take into consideration the opponent's reaction to our increased production. We will model the situation as a sequential game. First the FP decides whether to invest or not, then they play Cournot game. To solve this game we can use backwards induction. To find the payoffs, we have to calculate the Cournot Nash equilibrium of this game. To find it, we have to find the best response function for both companies. The best response functions for this example are:

$$Q_{FP}^{BR}(Q_{FP}, Q_{MG}) = \frac{534 - Q_{MG}}{2}$$

$$Q_{MG}^{BR}(Q_{FP}, Q_{MG}) = \frac{504 - Q_{FP}}{2}$$

When we solve this system of equations, we will get the following equilibrium quantities:

$$Q_{FP} = 188$$

$$Q_{MG} = 158$$

We can also solve it graphically:

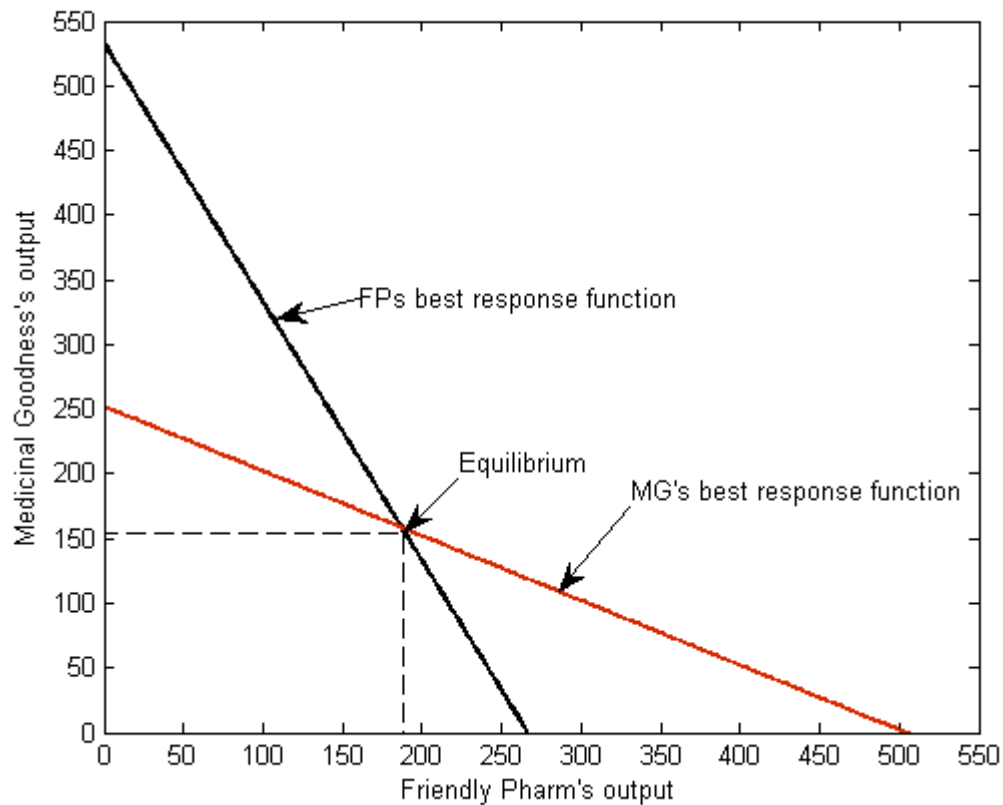


Figure 4.9 Graphical solution – „game theorist’s“

The price is now:

$$P = 109 - \frac{1}{6} * (188 + 158) = 51.333\text{€}$$

Now with the output of 188 kilograms the profit is:

$$\pi_{FP} = (51.333 - 20) * 188 = 5890.604$$

Now we see that the payoff from not investing in this new process is 4 704 000€ and the payoff from investing is 5 890 604€ - 1 000 000€ = 4 890 604€. The optimal decision is to invest in this new process. So, in conclusion, we advise to invest in this process as it will bring increased profit.

4.3 Role of uncertainty in decision making

In this chapter the necessary tools needed to treat games involving uncertainty will be recalled. First a concept of mixed strategies, strategies where each move is assigned a

probability, will be introduced. Later we will describe games with imperfect and incomplete information and tools needed to solve them.

4.3.1 Mixed strategies

Each strategy has to assign an action for each decision – which path to take. But this does not have to be defined absolutely. The action to be taken can be defined using probabilities – each action will be played with a certain probability. Let us explain the issue on the following game.

Two players named Alice (A) and Bob (B) are playing a game of rock-paper-scissors. The loser has to pay one euro to the winner. Bob brags to Alice that he is such a good player of this game, that if they tie with a rock, it will count as a win for her, and he will pay her one euro. The payoffs for this game are:

		Bob		
		Rock (q_1)	Paper (q_2)	Scissors (q_3)
Alice	Rock (p_1)	(1, -1)	(-1, 1)	(1, -1)
	Paper (p_2)	(1, -1)	(0, 0)	(-1, 1)
	Scissors (p_3)	(-1, 1)	(1, -1)	(0, 0)
Expected payoff to Bob		$-p_1 - p_2 + p_3$	$p_1 - p_3$	$-p_1 + p_2$

Table 4.1 Payoffs for the modified rock-paper-scissors

4.3.1.1 Solution with pure strategies

The pure strategies for this game are:

- Always play rock – Rock strategy
- Always play paper – Paper strategy
- Always play scissors – Scissors strategy

We can try finding pure strategy equilibrium for this game:

- If A plays Rock strategy, the best response from B is to play Paper. Given that B plays Paper, the best response from A is to play Scissors. {Rock, Paper} is not an equilibrium strategy profile.
- If A plays Paper strategy, the best response from B is to play Scissors. Given that B plays Scissors, the best response from A is to play Rock. {Paper, Scissors} is also not an equilibrium strategy profile.

This way we can check all of the possible strategy profiles and we will find out that there is no equilibrium for this game using only pure strategies.

4.3.1.2 Solution with mixed strategies

Now the players will choose each action with a certain probability – they will play **mixed strategies**. If the mixed strategies contain all of the player's pure strategies, they are **completely mixed strategies**. To find the optimal probabilities with which the player should play each action we have to calculate the expected payoffs first. Expected payoff of a strategy is the sum of the probability of each action multiplied by its payoff. The probabilities for player A playing rock, paper or scissors will be denoted p_1 , p_2 and p_3 respectively and for player B q_1 , q_2 and q_3 respectively. It is important that $p_1+p_2+p_3=1$, and $q_1+q_2+q_3=1$. The equations for the expected payoffs are in the following table:

		Bob			
		Rock (q_1)	Paper (q_2)	Scissors (q_3)	Expected payoff to Alice
Alice	Rock (p_1)	(1, -1)	(-1, 1)	(1, -1)	$q_1 - q_2 + q_3$
	Paper (p_2)	(1, -1)	(0, 0)	(-1, 1)	$q_1 - q_3$
	Scissors (p_3)	(-1, 1)	(1, -1)	(0, 0)	$q_1 - q_2 + q_3$
Expected payoff to Bob		$-p_1 - p_2 + p_3$	$p_1 - p_3$	$-p_1 + p_2$	
Payoffs: (Alice, Bob)					

Table 4.2 Expected payoffs for the modified rock-paper-scissors

After solving the system of equations we will get the following solution:

$p_1=1/3$, $p_2=4/9$, $p_3=2/9$, $q_1=1/3$, $q_2=2/9$, $q_3=4/9$. It means that player A will play rock with probability of 1/3, paper with probability of 4/9 and scissors with probability of 2/9 and respectively for player B.

This is the Nash equilibrium with mixed strategies. It means that if both players play the actions with those probabilities, they are both equally likely to win.

An important fact about games and mixed strategies is given by:

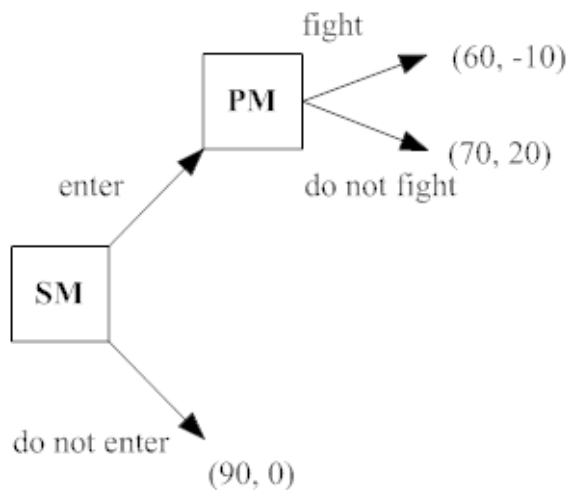
“Every game with a finite number of players, each of whom has a finite number of pure strategies, **possesses at least one Nash equilibrium**, possibly in mixed strategies.”

(BIERMAN and FERNANDEZ, 1998, p.220)

4.3.2 Games with imperfect information

To describe the games with imperfect information we employ the following example:

We have a monopolist market for smart phones. There is an incumbent firm - Pear Mobile (PM) and a firm that wants to enter this market - Small Mobile (SM). Both companies have perfect information about the game structure and payoffs. The game consists of two stages. In first stage the entrant decides whether to enter the market or not. If the entrant enters the market the game goes into second stage. In the second stage PM decides whether to fight the entrant and invest in a huge TV advertisement campaign or not fight. There are costs associated with entering the market. It is a dynamic game with perfect information. Payoffs are in millions of euro. The game tree is depicted in Figure 6.1.



Payoffs: (PM, SM)

Figure 4.10 Game tree for the entry deterrence game with perfect information

4.3.2.1 Solution with perfect information

We can find the solution for this game using the backwards induction. Given that SM will choose to enter, the best response for PM is to not fight them and get a payoff of 70. As both players have complete knowledge of the game, SM observes this and chooses to enter the market and gets payoff of 20. So the strategy profile {enter, do not fight} is the subgame perfect equilibrium for this game. If the incumbent firm says that

they are going to fight if the entrant enters, they are making an **incredible threat**, as it is not in their best interest to actually fight SM when they enter.

4.3.2.2 Solution with imperfect information

Now we will modify this example by stating that if SM enters and PM fights, there is a 20% chance that the ad campaign will be a great success. That means it will lure more customers and PM will make more money when fighting than when not fighting. It will be modeled by introducing the player Nature into the game which decides the outcome of the campaign if SM enters and PM decides to fight. Neither player knows the outcome of the campaign. This is a game with **imperfect information**. The game tree now looks like this (terminal nodes are labeled to make the next step clearer):

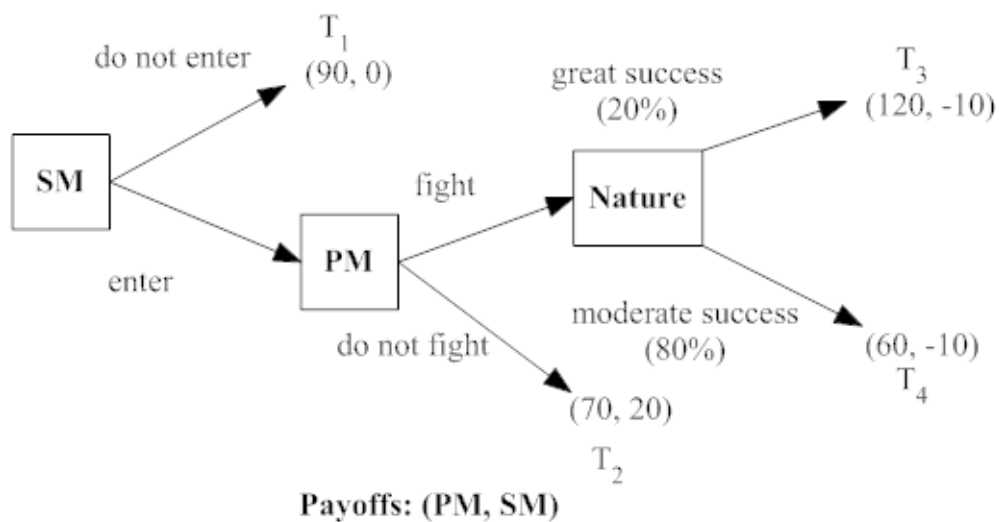


Figure 4.11 Game tree for the entry deterrence game with incomplete information

To solve this game with probabilities we need to modify the backwards induction algorithm to work with Nature's moves.

4.3.2.3 Modified backwards induction

We need to modify the first three steps of the backwards induction algorithm like this:

”

1. Start at the terminal nodes of the game and trace each one to its immediate predecessor, which will be a decision node for some player. These decision nodes are either “trivial”, “basic”, or “complex”. A decision node is **basic** if each of its branches you can either reach exactly one terminal node or a basic

decision node belonging to Nature. A basic node with only one branch is **trivial**. A decision node is **complex** if, starting from one of its branches, you can reach one or more decision nodes of strategic players or a complex decision node belonging to Nature. If you reach a trivial decision node, then keep moving up the tree until you reach either a complex or nontrivial basic decision node or until you can go no further.

2.

- a. If the basic node belongs to Nature, then when this node is reached, the outcome is determined at random using predetermined probabilities. Calculate the expected payoff for each player using these probabilities.
- b. If the basic node belongs to a strategic player, then find the optimal move by comparing the payoffs the player obtains at each terminal node reached from this decision node. Note that every path between a basic decision node A and a terminal node B starts at a unique branch of A. The branch that leads to the highest payoff for the player is the optimal move to make at that node.

3.

- a. Erase all the branches of the basic decision nodes belonging to Nature that you examined in step 2a. Each of these decision nodes now becomes a terminal node. Assign to these terminal nodes the expected payoffs you calculated in step 2a.
- b. Erase all the non-optimal branches that originate from each of the basic decision nodes you examined in step 2b. Each of these basic decision nodes becomes trivial. “

(BIERMAN and FERNANDEZ, 1998, p.212-213)

Now we apply this modified backwards induction algorithm to the entry deterrence game with incomplete information. When we come to the Nature's move, we calculate the expected payoff from terminal nodes T_4 and T_3 . The expected payoff for PM is $0.8 * 60 + 0.2 * 120 = 48 + 24 = 72$ and for SM it is $0.2 * (-10) + 0.8 * (-10) = -10$. After removing the branches belonging to Nature, the tree now looks like this:

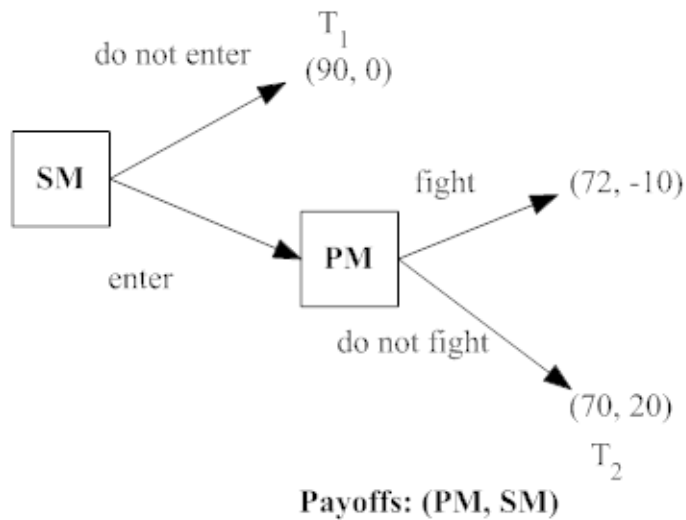


Figure 4.12 Pruned game tree for the entry deterrence game with incomplete information

Now we can see that PM's best move is to fight with a payoff of 72 compared to 70 when not fighting. After eliminating the corresponding branch, the tree now looks like this:

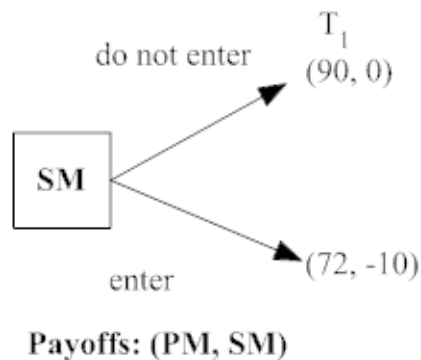


Figure 4.13 Game tree for the entry deterrence game with incomplete information after more pruning

From this game tree we can see that SM's optimal move is not to enter the market and get a payoff of 0, compared to -10 when entering. We have found the subgame perfect equilibrium and it is {fight, do not enter}.

4.3.3 Games with incomplete information

In this subchapter we will analyze and solve static and dynamic games with incomplete information.

4.3.3.1 Static games with incomplete information – static Bayesian games

Static game with incomplete information is called a Bayesian game. The normal form of the Bayesian game differs from the normal form of a game with complete information. It also involves player **types** and **prior beliefs**. The player type is information which each player knows about him, but not about other players and it determines his payoff function. In the case of previous example we can say that player PM can be of two types, one with successful campaign and the other one with a not successful campaign. Player's prior beliefs about other players' types represent the probabilities for each possible type of other players.

(GIBBONS, 1992)

We will modify the previous entry deterrence game again. This time PM has done extensive market research and knows for sure, what impact the campaign is going to have. One possible outcome is that the campaign will be a great success and PM will earn high profit. Another is that the campaign will be a failure and will let the competitor enter the market and make a profit. In this game PM moves simultaneously with SM and decides whether to invest in the campaign or not. At the same time SM independently decides whether to enter the market or not. SM did their research too, but they only know that with 60% probability the campaign will be a great success and with 40% probability a failure. It is a static game with incomplete information, also called a static Bayesian game. SM does not know if they are playing a game where the PM's campaign will be a great success or a game where PM's campaign will be a failure. PM knows this. The payoffs of these different games are given in Table 6.3.

		Great success		Failure	
		SM		SM	
		Enter	Don't enter	Enter	Don't enter
PM	Do campaign	(100, -10)	(120, 0)	(60, -10)	(80, 0)
	Don't do campaign	(70, 20)	(90, 0)	(70, 20)	(90, 0)
Payoffs: (PM, SM)					

Table 4.3 Payoffs for the static game with incomplete information

We can identify that in both of these games PM has a dominant strategy. In the game, where the campaign is a great success PM's dominant strategy is to Do campaign. In the game where the campaign is a failure Don't do campaign is PM's dominant strategy. SM does not have a dominant strategy. To solve this game we have to use the **Harsanyi transformation**.

Harsanyi transformation

The **Harsanyi transformation**, formulated by John Harsanyi in 1967 transforms a game with incomplete information into a game with complete but imperfect information. This transformation is done in this way: first the Nature decides the types of each player based on the probability of each type. Then Nature reveals to each player his own type. After that both players make their decisions simultaneously and receive their payoffs based on their types.

(GIBBONS, 1992)

To find the optimal solution for the game after the Harsanyi transformation we have to find the **Bayesian Nash equilibrium**.

Bayesian Nash equilibrium

In Bayesian games players try to maximize their **conditional expected utilities**. The conditional expected utilities are calculated for every strategy profile from the expected utilities which depend on types of players, multiplied by conditional probability calculated from prior beliefs about the players' types.

(BIERMAN and FERNANDEZ, 1998)

How to do the calculations is beyond the scope of this work. See the above mentioned literature for example.

"Concepts of dominant strategy (both strict and weak), dominant strategy equilibrium, iterated dominant strategy equilibrium, and Nash equilibrium all extend naturally to this new class of games if we replace the word "payoff" with "conditional expected utility"."

(BIERMAN and FERNANDEZ, 1998, p.278)

The extension of Nash equilibrium to Bayesian games is called the **Bayesian Nash equilibrium**.

The entry deterrence game can now be represented by the following table using the conditional expected utilities:

		SM	
		Enter	Don't enter
PM	(Do campaign, Do campaign)	((100, 60), (-10))	((120, 80), (0))
	(Do campaign, Don't do campaign)	((100, 70), (2))	((120, 90), (0))
	(Don't do campaign, Do campaign)	((70, 60), (8))	((90, 80), (0))
	(Don't do campaign, Don't do campaign)	((70, 70), (20))	((90, 90), (0))
Conditional Expected Utilities: ((successful PM, not successful), (SM))			

Table 4.4 Conditional expected utilities for the static game with incomplete information

Now we can find the strategy profile that is the Bayesian Nash equilibrium. If we evaluate all of the strategy profiles we can find that the strategy profile {(Do campaign, Don't do campaign), (Enter)} is the only Bayesian Nash equilibrium for this game.

4.3.3.2 Dynamic games with incomplete information

Again the entry deterrence game will be slightly modified. This time PM knows whether the campaign will be a great success or a failure and has to make the decision to do it or not. But the campaign is also more expensive. PM is of two types: Successful and Unsuccessful. SM has done some market research and has prior beliefs that with the probability of 25% the campaign will be a great success and with the probability of 75% it will be a failure. Players move in sequence, first PM has to decide whether to do the campaign or not. After that SM, observing PM's decision but not knowing their type, has to make a decision whether to enter the market or not. It is a dynamic game with incomplete information. We will use the Harsanyi transformation and model this game as a game with imperfect information where Nature moves first and decides the type of PM. The game tree with payoffs is depicted in Figure 6.5.

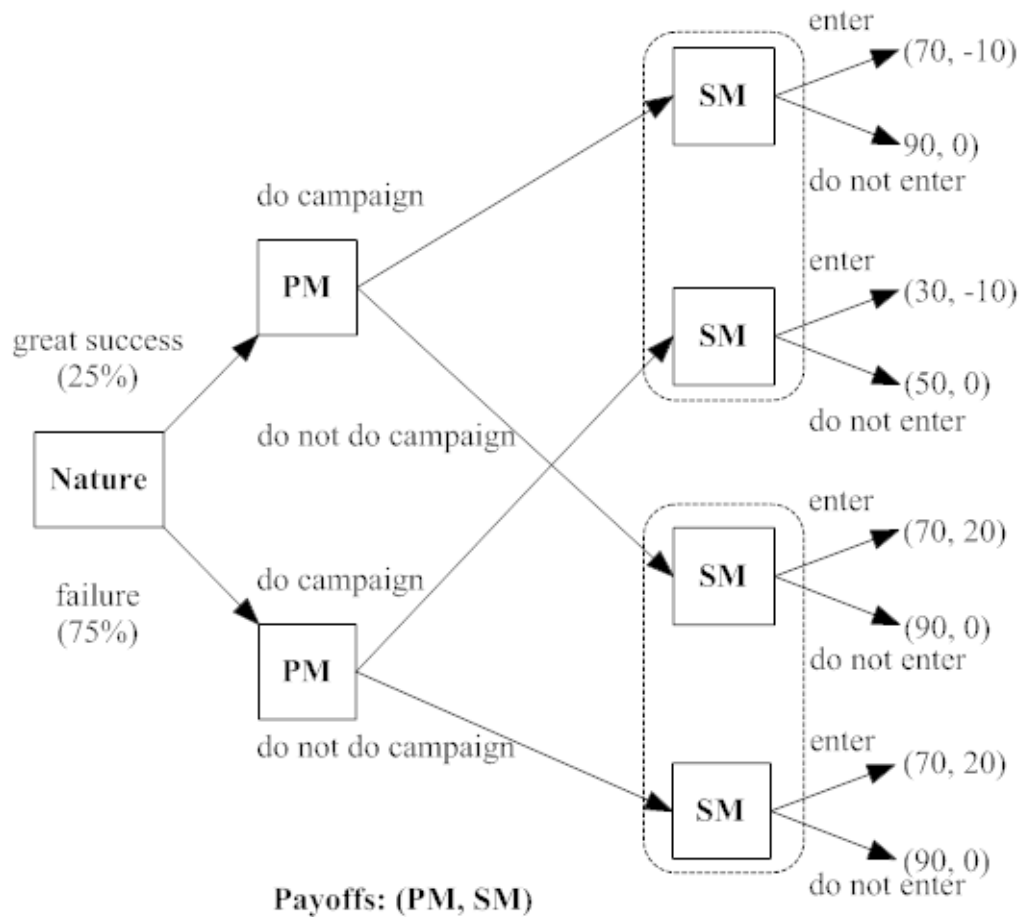


Figure 4.14 Game tree for the dynamic game with incomplete information

The dashed rectangles around SM's decision nodes represent two different **information sets**. At the time SM is making their decision to enter the market or not, they do not know the type of PM, they only know their previous decision to do the campaign or not. PM has two decisions to make: whether to do the campaign when they know it will be successful and whether to do the campaign when they know it will be a failure. SM has two decisions to make: whether to enter the market when PM does the campaign and whether to enter the market when PM does not do the campaign. Now we have to calculate the conditional expected payoffs and find the Nash equilibria of this game. The normal form representation of the game:

		SM	
		(Enter, Enter)	(Enter, Don't Enter)
PM	(Do campaign, Do campaign)	((70, 30), (-10))	((70, 50), (-2.5))
	(Do campaign, Don't do campaign)	((70, 70), (12.5))	((70, 90), (-2.5))
	(Don't do campaign, Do campaign)	((70, 30), (-2.5))	((70, 50), (5))
	(Don't do campaign, Don't do campaign)	((70, 70), (20))	((70, 90), (5))
		(Don't Enter, Enter)	(Don't Enter, Don't Enter)
	(Do campaign, Do campaign)	((90, 30), (-7.5))	((90, 50), (0))
	(Do campaign, Don't do campaign)	((90, 70), (15))	((90, 90), (0))
	(Don't do campaign, Do campaign)	((90, 30), (-7.5))	((90, 50), (0))
	(Don't do campaign, Don't do campaign)	((90, 70), (15))	((90, 90), (0))
Conditional Expected Payoffs: ((successful PM, not successful), (SM))			

Table 4.5 Normal form of the dynamic game with incomplete information

This game has two Nash equilibria: {(Don't do campaign, Don't do campaign), (Enter, Enter)}, {(Do campaign, Don't do campaign), (Don't enter, Enter)}. To find the one which does not involve incredible threats, we have to find the subgame perfect equilibrium. But in this game, the whole game is one subgame, so both strategy profiles are subgame perfect equilibria. We have to find the **perfect Bayesian equilibrium**.

Bayesian updating

Important notion is that SM does not know PM's type, but has some belief about it. SM assigns probabilities for each of the information sets (possible PM types) and this is called the **belief profile**. There are two nontrivial information sets in the previous example - Do campaign and Don't do campaign. Belief profile for these two information sets can be: {Do campaign: (successful: 0.50, unsuccessful: 0.50), Don't do campaign: ((successful: 0.50, unsuccessful: 0.50),)}. It means that SM believes that if PM does the campaign that they will be with 50% probability successful in campaign and with 50% unsuccessful (these beliefs are just for example). These probabilities have to be updated after SM observes PM's decision to reflect this decision. To update them we will use **Bayesian updating**. Bayesian updating uses the **Bayes theorem** to

calculate posterior probabilities (in the example it's the SM's belief about PM's type after observing PM's decision).

(BIERMAN and FERNANDEZ, 1998)

The calculations are beyond the scope of this work. See the above mentioned literature for example.

Perfect Bayesian equilibrium

"A perfect Bayesian equilibrium consists of a strategy profile and a belief profile such that:

1. The collection of strategies constitutes a Nash equilibrium, given the players' beliefs.
2. At each player's information set, the move required by the player's strategy maximizes that player's expected utility, given the player's beliefs about the state of the worlds at that information set and the other players' strategies.
3. Wherever possible, every player's beliefs can be derived from the equilibrium strategy profile and the common prior beliefs using Bayesian updating.

(BIERMAN and FERNANDEZ, 1998, p.328-329)

For the previous example the only perfect Bayesian equilibrium is the strategy profile {(Do campaign, Don't do campaign), (Don't enter, Enter))} and the belief profile {Do campaign: (successful: 1, unsuccessful: 0), Don't do campaign: ((successful: 0, unsuccessful: 1),)}. There is no belief profile for the other strategy profile to be a perfect Bayesian equilibrium. It involves incredible threat that SM will enter the market no matter what PM does, but it is not in their best interest to enter the market if PM does the campaign.

4.4 Auctions

Auctions are a quite common way of selling various types of things, where the potential buyers announce their prices and the seller (or the person that leads the auction) sells the item to the highest bidder. The buyers generally have different opinion about the value of the item being sold. Nowadays internet auction websites are a prospering trend. Also some of the governments use auctions to sell rights and properties and to raise funds.

We can distinguish these types of auction:

- Sealed bid auction – the bidders send their bids privately, bidders are not aware of how much other bidders bid. This type of bidding is commonly used for job contracts, where each potential contractor sends his bid in a closed envelope. The best bid wins the contract. The sealed bid auctions can have two variants:
 - First price auction – the winner of the auction pays the amount of money he bids
 - Second price auction – also called the Vickrey auction. The winner does not pay his own bid, but pays the second highest bid
- Open outcry auction – bidders openly announce their bids, for example they shout an amount of money they are willing to pay. The common variants are:
 - Ascending auction – also called English auction. It is a type of auction where the bidders make higher and higher bids and the last bidder-with the highest bid wins the auction
 - Descending auction – also called Dutch auction. In this type of auction the price starts at a certain level and descends with passing time. The price descends until someone makes a bid or it reaches the limit price set by the seller. The first bidder wins the auction and pays the current price. This type of auction was typically used to sell tulips in Netherlands.
- Common value auction – the real value of the item being sold is the same for all of the potential buyers, for example the drilling right in a certain location have the same value-the amount of gas there is in this location. But this value is not 100% known to everyone before the auction ends. Otherwise it would make no sense to sell it this way. Buyers have different assessments of the real value of the item being sold based on their beliefs, experience or previous research.
- Private value auction – the real value of the item being sold is different for each buyer. The buyer knows his evaluation of the object being sold but does not know the evaluations of other buyers. For example the value of the painting is different for people with different artistic tastes or family ties to the author of the painting.

- Single-unit auction – in this auction there is only one item being sold. For example a sale of one house.
- Multi-unit auction – in this auction there is more than one identical item being sold, and the buyers not only announce their bid, but also the quantity they want to buy.

In the following subchapters we will explain the terms needed and compare the monetary gain for the seller with different types of auctions.

4.4.1 Winner's curse

Consider the following example:

You want to buy a used boat in an Internet auction. It is an open outcry ascending (English) common value auction. All of the potential buyers make public bids and the highest bid wins the auction. The real value of the boat being sold is the same for each buyer - it is 10 000€, but their predictions of this value differ. The seller is only going to sell the boat if the offer is higher than 9000€, which is his **reserve price**. The value of the boat is different for the seller and for the buyers. We can for example say that there are 5 other potential buyers and they value the boat at 9 800€, 9 900€, 10 000€, 10 100€ and 10 200€ respectively.

If all of these buyers bid their true evaluations for the boat, the fifth bidder would win the auction with the bid of 10 200€ and lose 200€ on it, as the boat is only worth 10 000€. That is called the **winner's curse**. Winner's curse is very common in real world common value auctions. In private value auctions each buyer knows the real value of the object to him, so the winner's curse is not a problem.

4.4.2 Auction as a game

If the bidders don't always bid their true evaluations, we call it **bid shading**. The auction, where buyers (players) are rational and play strategically (they shade their bids), can be thought of as a game and analyzed using the tools of game theory.

4.4.3 Comparison of auction types with perfect information

Consider the following example:

There is an auction selling a flat. There are only two buyers - Andrew (A) and Bill (B). Player A's evaluation of the flat is $v_A = 58\,000\text{€}$ and player B's evaluation is $v_B = 42\,000\text{€}$. The price of the flat will be denoted as P . The price and also the payoffs will be in thousands of euro. The payoff for player A if he wins the auction is equal to $v_A - P$. The payoff for player B if he wins the auction is $v_B - P$. The payoff for the losing player is 0. The bids of individual players will be denoted b_A and b_B respectively.

4.4.3.1 First price sealed bid auction

The players announce their bids simultaneously and independently. Winner is the player who made the highest bid and he pays his bid. If both players bid the same, the winner is decided by a toss of a coin, giving each player a 50% chance of winning. This auction is a static game with perfect information. The bids can only be made in multiples of $10\,000\text{€}$. The strategic form of the game is described by Table 7.1.

		Andrew				
		10	20	30	40	50
Bill	10	(16, 24)	(0, 38)	(0, 28)	(0, 18)	(0, 8)
	20	(22, 0)	(11, 19)	(0, 28)	(0, 18)	(0, 8)
	30	(12, 0)	(12, 0)	(6, 14)	(0, 18)	(0, 8)
	40	(2, 0)	(2, 0)	(5, 0)	(1, 9)	(0, 8)
Expected payoffs: (Bill, Andrew)						

Table 4.6 Strategic form of the first price sealed bid auction

Since Andrew knows that his evaluation of the flat is higher than Bill's evaluation he is going to bid only as low as possible higher amount of money than Bill. Because the bids have to be multiples of $10\,000\text{€}$, he will bid $50\,000\text{€}$ and win the auction. This solution is the only iterated weakly dominant equilibrium. If the bids can be made in multiples of one euro, Andrew would bid only one euro higher than Bill's evaluation of the flat ($42\,001\text{€}$) and win the auction. If the increments between bids approach zero, winner is Andrew, because he values the flat the most, and he pays $42\,000\text{€}$ - Bill's evaluation of the flat.

4.4.3.2 Second price sealed bid auction

The players announce their bids simultaneously and independently. Winner is the player who made the highest bid and he pays what the loser bid. If both players bid the same,

the winner is decided by a toss of a coin, giving each player a 50% chance of winning. The game may be described by Table 7.2.

		Andrew				
		10	20	30	40	50
Bill	10	(16, 24)	(0, 48)	(0, 48)	(0, 48)	(0, 48)
	20	(32, 0)	(11, 19)	(0, 38)	(0, 38)	(0, 38)
	30	(32, 0)	(22, 0)	(6, 14)	(0, 28)	(0, 28)
	40	(32, 0)	(22, 0)	(12, 0)	(1, 9)	(0, 18)
		Expected payoffs: (Bill, Andrew)				

Table 4.7 Strategic form of the second price sealed bid auction

There is more than one equilibrium solution for this game. Important equilibrium is the one where both players bid their true evaluations $b_A=50$ and $b_B=40$. It shows that it is rational to bid the true evaluation in second price auctions. Again, in this auction if the increments between bids approach zero, winner is Andrew, because he values the flat the most, and he pays 42 000€- Bill's evaluation of the flat.

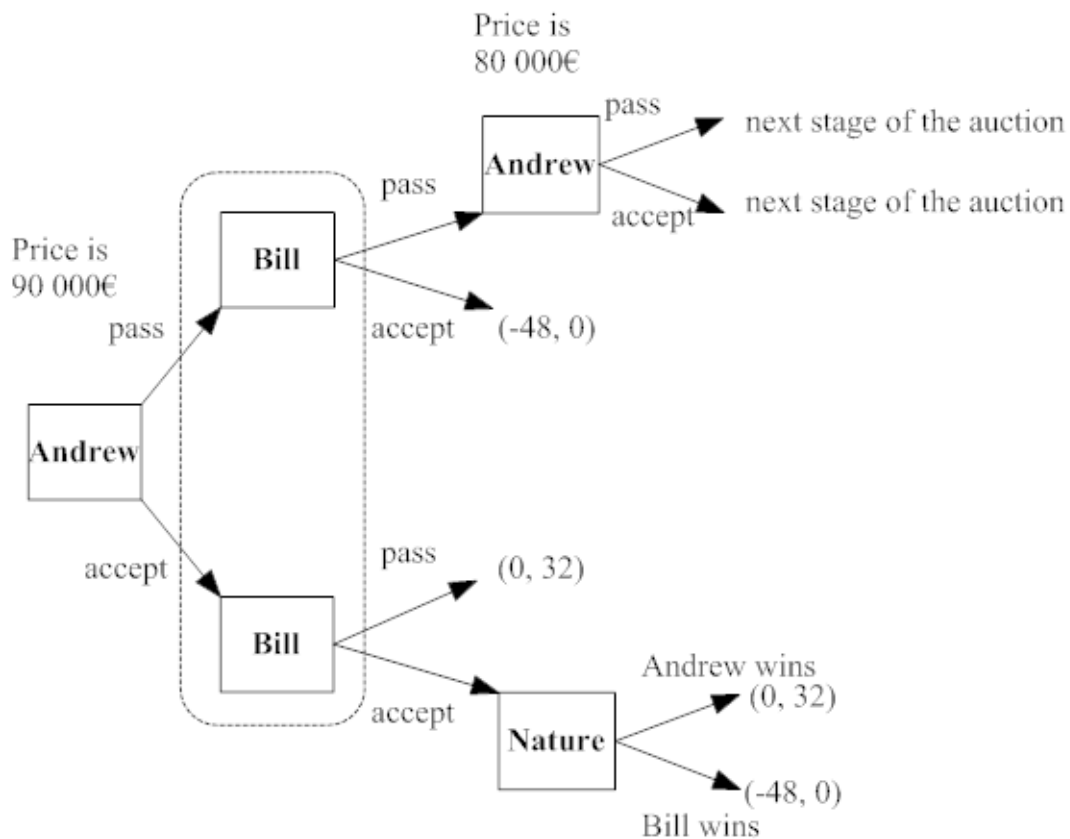
4.4.3.3 Open outcry ascending (English) auction

In this auction both players alternately raise the price of the flat until the price is equal to Bill's evaluation. At this point Andrew only raises the price once more and wins the auction. If the increments approach zero he will pay Bill's evaluation of the flat. Again the player that valued the flat the most won the auction and paid the loser's evaluation.

4.4.3.4 Open outcry descending (Dutch) auction

In this auction, the price begins at for example 90 000€ Then this price gets lowered each time by 10 000€ until one of the players accepts the price. When one of the players accepts the announced price, he wins the auction, pays the price and the auction ends. If both players accept the price at the same time, the winner is decided by the flip of a coin (Nature). When the price drops to 0€, the auction ends without selling the flat.

This game is sketched in Figure 7.1.



Payoffs: (Bill, Andrew)

Figure 4.15 Part of the game tree for the open outcry descending auction

The dashed rectangle around Bill's decision nodes means that they are in the same information set - it models that at the time that Bill is making his decision to pass or accept he does not know Andrew's decision.

In this auction Bill's strategy is to pass until the price drops to 40 000€ Andrew knows this, so his strategy is to pass until the price drops to 50 000€ and accept that price. Andrew will win the auction and pay 50 000€ If the decrements approach zero, Andrew will win because he values the flat the most and he will pay Bill's evaluation.

4.4.3.5 Summary

For all four types of auctions the result is the same. The player with the highest evaluation of the item being sold wins the auction and he pays the evaluation of the loser. The revenue for the seller is the same too.

4.4.4 Comparison of auction types with imperfect information

Consider the following example:

There is a small art auction, selling a few works of a recently deceased local artist. There are two buyers interested in one of the paintings being sold. One of them - Mr. Stein (S) is a family member of the artist that painted the particular painting. For him the value of the painting is V_S and his bid will be denoted b_S . The second buyer is an art collector Mr. Hoarder (H) and the painting has V_H value for him and his bid will be denoted b_H . Both players believe that the other one values the painting between 0€ and 100€ with uniform distribution. The players' valuations are independent of each other and not known to other players. It is an **independent private value auction**.

4.4.4.1 First price sealed bid auction

Players simultaneously make their bid which means this is a static Bayesian game. The winner is the player with the highest bid and he pays his bid. The actual calculations of the Bayesian Nash equilibrium are beyond the scope of this work. The solution is that both players will bid half of their evaluation. Expected revenue of the seller is:

$$R = E\left(\frac{1}{2} \max\{V_S, V_H\}\right) = \frac{1}{2} \int_0^{100} \int_0^{100} \max\{V_S, V_H\} dV_S dV_H = \frac{100}{3}$$

(BIERMAN and FERNANDEZ, 1998, p.299)

So the expected revenue is 333.33€ The highest bidder wins the auction and pays his bid.

4.4.4.2 Second price sealed bid auction

In this auction the winner pays the loser's bid. In equilibrium in this auction both players bid their true evaluations. It is a dominant strategy for both players. The revenue for the seller is:

$$R = E(\min\{V_S, V_H\}) = \frac{1}{2} \int_0^{100} \int_0^{100} \min\{V_S, V_H\} dV_S dV_H = \frac{100}{3}$$

(BIERMAN and FERNANDEZ, 1998, p.301)

So the expected revenue is 333.33€ The highest bidder wins the auction and pays his opponent's bid.

4.4.4.3 The revenue equivalence theorem

“In an independent private values environment with risk-neutral bidders, the expected price paid for the object is the same under the first-price and second-price sealed bid auctions.”

(BIERMAN and FERNANDEZ, 1998, p.301)

This theorem states that it does not matter which type of auction we use, the seller makes the same amount of money. In practice, the results are different because of various other factors.

4.4.4.4 Open outcry ascending (English) auction

“The Dutch auction is strategically equivalent to the first price, sealed bid auction, whatever the auction environment.”

(BIERMAN and FERNANDEZ, 1998, p.302)

This theorem states that the open outcry ascending auction will yield the same equilibrium outcome as the first price, sealed bid auction, as the players' strategies are the same.

4.4.4.5 Open outcry descending (Dutch) auction

If the bidder's valuations are not correlated and are initially known, the following is true for the open outcry descending auctions:

“In an independent private values environment, the English auction is strategically equivalent to the second price, sealed bid (“Vickrey”) auction.”

(BIERMAN and FERNANDEZ, 1998, p.302)

This theorem states that the open outcry descending auction will yield the same equilibrium outcome as the second price, sealed bid auction, as the players' strategies are the same.

4.4.4.6 Summary

In all types of auctions mentioned before the revenue to the seller is the same. The player with the highest bid for the item being sold wins the auction.

4.5 Computer science and game theory

In this chapter a few examples of applications of game theory which are related to computer science and information technology will be presented. It is the subject of study of Algorithmic and Computational game theory.

Algorithmic game theory is a science field which analyzes and designs algorithms in strategic environment using game theoretic tools. Closely related to it is the computational game theory which uses computer systems to solve and design game theoretic models and also uses game theory to model computer systems and network. Game theory is also applied in the field of multi-agent systems and other fields.

4.5.1 Languages, libraries and simulation tools

There are libraries of game theoretic tools and software for programming languages including C, C++, Java. Some example implementations:

- Gambit – a library of game theoretic tools written in C++
- Trade Network Game – a framework for simulating trade networks written in C++

Game theoretic models can be simulated using a variety of simulation tools including:

- Java applets on the internet for simple simulations
- Matlab toolboxes

There are many specialized pieces of software that predict the behavior in economy, politics and other fields using simulations of the game theoretic models.

4.5.2 Routing

Game theory can be used to analyze the problem of routing in computer networks. For example we can take the following network:

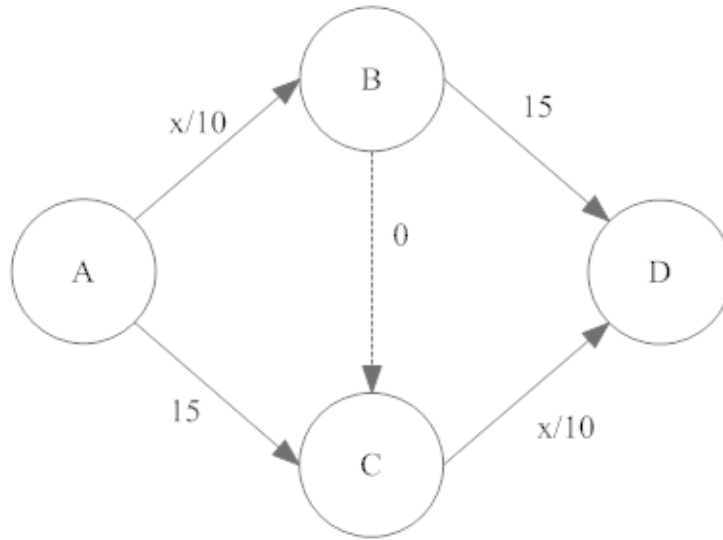


Figure 4.16 Diagram of a network illustrating the Braess's paradox

The (unregulated) network is given as four connected nodes with given rate of traffic of 200 packets sent from A to D. The goal is to assign traffic to paths in a way which minimizes total latency in the network. The latency at each path is given in the graph (x is number of packets sent through the node). At each node, the path with the lowest latency is selected – the players are rational (selfish). The Nash equilibrium solution always has the same latency for each chosen path from start node (A) to end node (D). If we for the moment ignore the path from B to C which has latency 0, the Nash equilibrium solution for this game is that $200/2=100$ packets will go through the node B and 100 packets will be routed through C and the total latency in the network will be

$$L = \frac{\frac{200}{2}}{10} + 15 = 25$$

This is also the optimal solution.

When we consider the path between nodes B and C which has latency 0, the whole traffic will be routed on the A->B->C->D path and the latency will be:

$$L = \frac{200}{10} + 0 + \frac{200}{10} = 40$$

There is a significant increase in total latency when we add the B->C path. This is an example of the **Braess's paradox**. It is a Nash equilibrium solution but not an optimal one. The optimal solution would be the same as before, that is to route half of the traffic through A->B->D and half through A->C->D.

(AXELROD and WERNER, 2004)

4.5.3 Online advertisements and auctions

Auctions are widely used on the internet to sell various products and online advertisement. The best known examples are probably Google and eBay.

Google is a web search engine which generates most of the revenue with sponsored text advertisements. In Google they use a second price auction to determine the price for the sponsored search results. The advertisers compete for advertising slots in the search results by posting the maximum price they are willing to pay for a click or thousand impressions. The auctions are completely automatic and take place every time a user searches for keywords in which the advertisers are interested. Winners are ranked considering a number of factors including the price the advertiser is willing to pay (his bid) and the quality of the advertisements - auction. The winning advertisements are presented to the user. If the user clicks on the advertisement, the advertiser is charged according to the next highest ranking advertisement.

(GOOGLE, 2012), (VARIAN, 2008)

eBay is an online auction website and it uses second price auctions to sell various products. The sellers post items for sale and they are able to specify their reservation price and limit the length of the auction. The buyers post their bids for the items they want to buy and the highest bidder wins the auction. The winner then pays the next highest bid.

4.5.4 Peer-to-peer systems

Peer-to-peer systems are most known for the file-sharing service like Napster or BitTorrent. In peer to peer systems users usually assume two roles: a client that wants to download a file (use a service) or a server that uploads this file (provides the service). In order to achieve stability in an environment with many users it is necessary to design the system in a way to encourage cooperation between users and discourage “free-riding” – downloading and not uploading. This is also applicable to problems of inter-domain routing, community wireless networks and many others. Game theory is applied to design such mechanisms. The players are rational and try to maximize their payoffs. Their strategy is how much they are willing to contribute. The game played here is a

repeated game with many players. The goal is to find equilibrium solution where users cooperate.

One possible solution is to use reputation and service differentiation based systems. In these systems users that contribute to the system are given better service than the users that do not contribute to the system. Their reputation is based on the overall contribution (ratio between upload and download). This creates incentives for the users to contribute to the system (upload more than download).

Another solution is used by the BitTorrent, which uses multiple interactions between the same users when downloading one file. They achieve this by splitting the files into many small parts, which the peers share between themselves.

Other solutions are based on using currency which is gained by contributing.

(NISAN (ed.), et al., 2007)

4.5.5 Other applications

Game theory has also been applied to problems in:

- Cryptography and information security
- Load balancing
- Network design
- Resource allocation

5 Conclusion

This work provides a survey of the most commonly used game theoretic models and their economic applications. It involves a review of basic game theoretic tools, modeling oligopolies, the role of information in decision making, auctions and a few computer science related applications.

Game theory is a mathematical discipline claiming high professional mathematical background, which is not easy to grasp for nonmathematicians seeking this discipline for applications. It was imperative to understand the theoretic principles to formulate the models used in this work.

Almost all of the sources used for this work are in English as there is a shortage of bibliography in Czech/Slovak language.

My own contribution was mainly getting familiar with the tools of game theory and creating and solving the models for the presented economic examples.

Game theoretic tools are being used not only in economics but also in biology, political science, philosophy, computer science and others. Recently, game theory is being applied on a broader scale. There are many other important applications related to economics not included in this work. Most notably: repeated and cooperative games, voting, effects of moral hazard and the field of evolutionary game theory.

Game theory and its applications still promise a lot more in the future.

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